NB10 ADVANCED – CENTRE OF MASS AND COLLISIONS TWT

SOLUTIONS

1. Assume that initial angular momenta of two particles are p_1 and p_2 and the angle between them is θ.

   Then, $|p_1| = mv$ and $|p_2| = mv$

   If final momentum is p then,

   $|p| = (2m)(v/2) = mv$

   From law of conservation of momentum,

   $\vec{p} = \vec{p}_1 + \vec{p}_2$

   Hence, $|p| = \sqrt{p_1^2 + p_2^2 + 2p_1p_2 \cos\theta}$

   So we get, $mv = \sqrt{(mv)^2 + (mv)^2 + 2(mv)(mv)\cos\theta}$

   So $\cos\theta = -1/2$ ➔ $\theta = 120^0$

   Answer : Option (C)

2. **Answer : Option (C)**

3. There are no horizontal forces acting on the hemisphere, as the surfaces are smooth. As CoM is having zero initial velocity in horizontal direction, it will not move in horizontal direction. In vertical direction, the force of gravity will pull the CoM down. So it will follow a vertical straight line.

   Answer : Option (C)

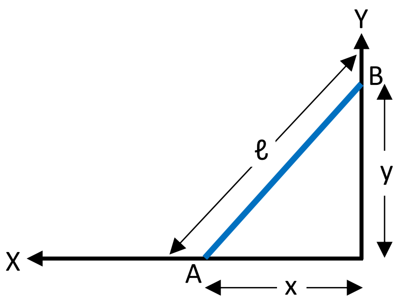
4. We have,

   $x^2 + y^2 = \ell^2$

   Differentiating both the sides w.r.t. time,

   $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

   Hence, $\frac{dy}{dt} = V_y = -\frac{x}{y}V_x = -\frac{3}{4}(2) = -1.5\ ms^{-1}$.

   V_{CM} = Velocity of center of rod = $\frac{V_A+V_B}{2} = \frac{2\hat{\imath} - 1.5\hat{\jmath}}{2}$

   $|V_{CM}| = 1.25\ ms^{-1}$

   Answer : Option (A)

5. From the equation of rocket propulsion,

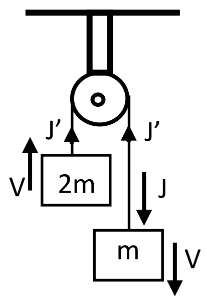
   $V_{rel}\,dm = -m\,dV$ ➔ $dV = -V_{rel}\frac{dm}{m}$

   $\int_0^V dV = -V_{rel}\int_{m_i}^{m_f}\frac{dm}{m}$ ➔ $V = V_{rel}\ \ell n\frac{m_i}{m_f} = 10\ \ell n\frac{40}{38}$

   Answer : Option (D)

6. **Answer : Option (A, B, D)**

7. Assume impulse along the string is J'.



Also assume that common velocity of the blocks is V.

J – J' = mV .... (Right block)

J' = 2m V .... (Left block)

Adding these two equations,

J = 3mV ➔ $V = \frac{J}{3m}$

Also, as heavier block is moving up with the same speed with which lighter block is moving down, center of mass must be moving up.

Answer : Option (A, D)

8. Take upwards as positive direction of Y-axis.

Acceleration of both the particles is $-g\,\hat{j}$

$a_{CM} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{m_1\,(-g\,\hat{j}) + m_2\,(-g\,\hat{j})}{m_1 + m_2} = -g\,\hat{j}.$

$V_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(2)(0)+(1)(4\,\hat{j})}{2+1} = \frac{4}{3}\hat{j}\,.$

As initial velocity of center of mass is positive but acceleration is negative, it will first go up and then come down.

Answer : Option (B, C)

9. As block of mass m_2 just starts moving when elongation is x, we can write,

Force of friction = Force due to spring

$\mu\, m_2\, g = kx$ ➔ $x = \frac{\mu\, m_2 g}{k}$

Now applying law of conservation of energy,

Work done by F = Work done by friction on block m_1 + Potential energy stored in the spring.

$Fx = \mu\, m_1\, g\, x + \frac{1}{2}k\, x^2$ ➔ $F = \mu\, m_1\, g + \frac{1}{2}kx = \mu\, m_1\, g + \frac{1}{2}\mu\, m_2\, g$

Answer : Option (B, C)

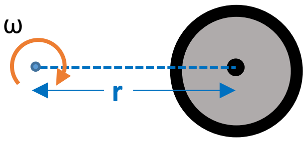
10. When the block reaches maximum height, it will come to rest w.r.t. wedge. Now from ground frame both of them may have some common horizontal velocity. But initial momentum of system in horizontal direction is zero. Also there are no forces in horizontal direction. So final momentum in horizontal direction must also be zero. So both the blocks will be at rest in ground frame. From law of conservation of energy, we can say that the block will rise to same height i.e. H = R.

Answer : Option (C, D)

11. **Answer : Option (B, D)**

12. **Answer : Option (B, C, D)**

13. We can assume that the mass of the ring is concentrated at the center of the ring.



Answer : Option (A, C, D)

14. During the collision, part of the energy of the ball goes into elastic deformation energy of the ground.

So **during** collision, mechanical energy of the ball may not be conserved.

As collision is elastic, no energy is lost (in the form of heat or sound) and so total energy of the (Ball + Earth) system remains same.

Answer : Option (C, D)

15. From the graph it can be seen clearly that collision takes place between t = 1 to t = 3.

Also, after collision, both R and S are having positive velocities means they are moving in same direction.

At the mid time of collision, i.e. at t = 2, both R and S are having common velocity.

Also, change in velocity of particle S is 1 ms^{-1} while that in case of particle R is 0.6 ms^{-1}. But we know that during collision, change in momentum (magnitude-wise) is same for both the particles. So one having higher change in velocity will have lower mass as change in momentum is same.

Answer : Option (A, B, C)

16. Assume that given object is made up of two discs – one having radius 'a' and mass '4M' and other having radius 'a/2' and mass '-M'.

$$(X_{CM}, Y_{CM}) = \frac{m_1\,(x_1, y_1) + m_2\,(x_2, y_2)}{m_1 + m_2} = \frac{4M\,(0,0) + (-M)(\frac{a}{2}, 0)}{4M + (-M)} = \left(-\frac{a}{6}, 0\right)$$

Answer : 6

17. Assume that coefficient of restitution is 'e'. Then the ball will strike the wall with velocity 'V' and return back with velocity 'eV'.

Time for 'to' motion = $\frac{d}{V}$

Time for 'fro' motion = $\frac{d}{eV}$

Avergage speed = $\frac{Total\ distance}{Total\ time} = \frac{2d}{\frac{d}{V} + \frac{d}{eV}} = \frac{2eV}{e+1}$

Hence, $\frac{2}{3}V = \frac{2e}{e+1}V$ ➔ e = 0.5

Answer : 5

18. Let the area of cross-section of the cylinder be A,

its mass be M and its initial velocity be V_0.

Assume that the front end of the cylinder has

already covered distance 'x' in dust cloud.

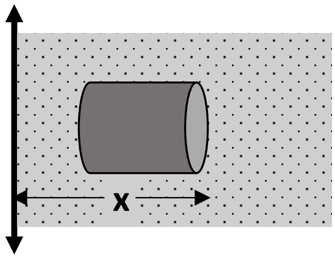
Assume that the velocity of the cylinder at this moment is 'V'.

Till now cylinder has collected dust particles from volume Ax.

Mass of dust collected is Ax ρ.

So total mass of the (Cylinder + Dust) = M + ρ A x

Then from law of conservation of momentum,



$MV_0 = (M + \rho A x)V = (M + \rho A x)\frac{dx}{dt}$

Hence, $MV_0\, dt = (M + \rho A x)\, dx$

$\int_0^T MV_0\, dt = \int_0^X (M + \rho A\, x)\, dx$ ➔ $MV_0 T = MX + \rho A X^2/2$

Answer : 1

19. As the collision is elastic and the blocks have same mass, they will exchange the velocities.

So block B will get velocity of 10 ms^{-1} after the collision.

From Work-Energy theorem,

Loss in KE of B = Work done by the friction.

$\frac{1}{2} mv^2 = \mu\, mg\, X$ where X is the distance covered by B before coming to rest.

Answer : 5

20. Assume that both the particles get velocity V along the string due to impulse.

Also assume that the impulse along the string is J.

Then, $10 \cos 30^0 - J = mV$

$J = mV$

Hence, $10 \cos 30^0 = 2J$

Answer : 5