JEE MAIN TEST SERIES

SOLUTIONS PART TEST-1

PART A – PHYSICS

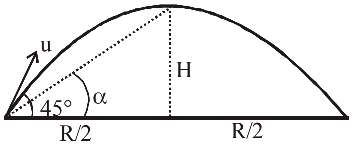
1. **(2)** $H = \dfrac{u^2 \sin^2 45°}{2g} = \dfrac{u^2}{4g}$ ...(1)

$R = \dfrac{u^2 \sin 90°}{g} = \dfrac{u^2}{g}$

or $\dfrac{R}{2} = \dfrac{u^2}{2g}$ ...(2)

$\therefore \tan\alpha = \dfrac{H}{R/2}$

$= \dfrac{\frac{u^2}{4g}}{\frac{u^2}{2g}} = \dfrac{1}{2} \quad \Rightarrow \alpha = \tan^{-1}\left(\dfrac{1}{2}\right)$



2. **(3)** The block begins to slide if
F cos 37° = μ (mg – F sin 37°)
5t [cos 37° + μ sin 37°] = μ mg

$5t\left[\dfrac{4}{5} + \dfrac{3}{5}\right] = 70$ or $t_0 = 10$ sec. or $t_0/2 = 5$ sec

3. **(3)** From law of conservation of angular momentum

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

Angular velocity of system $\omega = \dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

$\therefore$ Rotational kinetic energy $= \dfrac{1}{2}(I_1 + I_2)\omega^2$

$\dfrac{1}{2}(I_1 + I_2)\left(\dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2 = \dfrac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$

4. **(4)** Range of projectile, $R = \dfrac{u^2 \sin 2\theta}{g}$

If u and θ are constant then $R \propto \dfrac{1}{g}$

$\dfrac{R_m}{R_e} = \dfrac{g_e}{g_m} \Rightarrow \dfrac{R_m}{R_e} = \dfrac{1}{0.2} \Rightarrow R_m = \dfrac{R_e}{0.2} \Rightarrow R_m = 5R_e$

5. **(4)** $f = c\, m^x k^y$;

Spring constant, k = force/length.

$[M^0L^0T^{-1}] = [M^x (MT^{-2})^y] = [M^{x+y} T^{-2y}]$

$\Rightarrow x + y = 0, -2y = -1$ or $y = \dfrac{1}{2}$

Therefore, $x = -\dfrac{1}{2}$

6. **(3)** Given that $F(t) = F_0 e^{-bt}$

$\Rightarrow m\dfrac{dv}{dt} = F_0 e^{-bt}$

$\dfrac{dv}{dt} = \dfrac{F_0}{m} e^{-bt}$

$\int_0^v dv = \dfrac{F_0}{m}\int_0^t e^{-bt} dt$

$v = \dfrac{F_0}{m}\left[\dfrac{e^{-bt}}{-b}\right]_0^t = \dfrac{F_0}{mb}\left[-\left(e^{-bt} - e^{-0}\right)\right]$

$\Rightarrow v = \dfrac{F_0}{mb}\left[1 - e^{-bt}\right]$

7. **(2)** $W_A = \dfrac{1}{2}k_A x^2$ and $W_B = \dfrac{1}{2}k_B x^2$

As, $k_A > k_B$ and so $W_A > W_B$,

Also $W = \dfrac{1}{2}kx^2 = \dfrac{F^2}{2k}$

As, $k_A > k_B \therefore W_A' < W_B'$.

8. **(2)** $g' = g - \omega^2 R\cos^2\lambda$

To make effective acceleration due to gravity zero at equator $\lambda = 0$ and $g' = 0$

$\therefore 0 = g - \omega^2 R \Rightarrow \omega = \sqrt{\dfrac{g}{R}} = \dfrac{1}{800}\dfrac{rad}{s}$

9. **(1)** When ball collides with the ground it loses its 50% of energy

$\therefore \dfrac{KE_f}{KE_i} = \dfrac{1}{2}$

$\Rightarrow \dfrac{\frac{1}{2}mv_f^2}{\frac{1}{2}mv_i^2} = \dfrac{1}{2}$

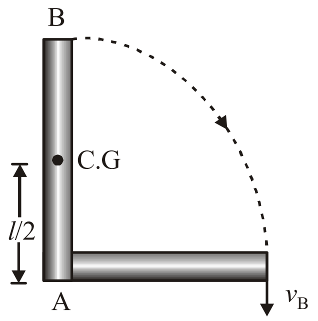
or $\dfrac{v_f}{v_i} = \dfrac{1}{\sqrt{2}}$

or, $\frac{\sqrt{2gh}}{\sqrt{v_0^2 + 2gh}} = \frac{1}{\sqrt{2}}$

or, $4gh = v_0^2 + 2gh$

$\therefore$ $v_0 = 20 ms^{-1}$

10. (2) In this process potential energy of the metre stick will be converted into rotational kinetic energy.



P.E. of meter stick $= mg\left(\frac{l}{2}\right)$

Because its centre of gravity lies at the middle point of the rod.

Rotational kinetic energy $E = \frac{1}{2}I\omega^2$

I = M.I. of metre stick about point $A = \frac{ml^2}{3}$

ω = Angular speed of the rod while striking the ground

v_B = Velocity of end B of metre stick while striking the ground

By the law of conservation of energy,

$$mg\left(\frac{1}{2}\right) = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{ml^2}{3}\left(\frac{v_B}{l}\right)^2$$

By solving we get, $v_B = \sqrt{3gl} = \sqrt{3\times10\times1} = 5.4\ m/s$

11. (2) Weight in air = (5.00 ± 0.05) N

Weight in water = (4.00 ± 0.05) N

Loss of weight in water = (1.00 ± 0.1) N

Now, relative density = $\frac{\text{Weight in air}}{\text{Weight loss in water}}$

i.e. Relative density = $\frac{5.00 \pm 0.05}{1.00 \pm 0.1}$

Now relative density with max permissible error

$$= \frac{5.00}{1.00} \pm \left(\frac{0.05}{5.00} + \frac{0.1}{1.00}\right)\times 100 = 5.0 \pm (1+10)\%$$

$$= (5.0 \pm 11\%)$$

12. (3) Here, $f = f_0\left(1 - \frac{t}{T}\right)$ or, $\frac{dv}{dt} = f_0\left(1 - \frac{t}{T}\right)$

or, $dv = f_0\left(1 - \frac{t}{T}\right)dt$

$\therefore$ $v = \int dv = \int\left[f_0\left(1 - \frac{t}{T}\right)\right]dt$

or, $v = f_0\left(t - \frac{t^2}{2T}\right) + C$

where C is the constant of integration.

At $t = 0$, $v = 0$.

$\therefore$ $0 = f_0\left(0 - \frac{0}{2T}\right) + C \quad \Rightarrow C = 0$

$\therefore$ $v = f_0\left(t - \frac{t^2}{2T}\right)$

If $f = 0$, then

$0 = f_0\left(1 - \frac{t}{T}\right) \Rightarrow t = T$

Hence, particle's velocity in the time interval $t = 0$ and $t = T$ is given by

$$v_x = \int_{t=0}^{t=T} dv = \int_{t=0}^{T}\left[f_0\left(1 - \frac{t}{T}\right)\right]dt$$

$$= f_0\left[\left(t - \frac{t^2}{2T}\right)\right]_0^T$$

$$= f_0\left(T - \frac{T^2}{2T}\right) = f_0\left(T - \frac{T}{2}\right)$$

$$= \frac{1}{2}f_0 T.$$

13. (3) According to problem disc is melted and recasted into a solid sphere so their volume will be same.

$$V_{Disc} = V_{Sphere} \Rightarrow \pi R_{Disc}^2 t = \frac{4}{3}\pi R_{Sphere}^3$$

$$\Rightarrow \pi R_{Disc}^2\left(\frac{R_{Disc}}{6}\right) = \frac{4}{3}\pi R_{Sphere}^3 \quad \left[t = \frac{R_{Disc}}{6},\ \text{given}\right]$$

$$\Rightarrow R_{Disc}^3 = 8R_{Sphere}^3 \Rightarrow R_{Sphere} = \frac{R_{Disc}}{2}$$

Moment of inertia of disc

$$I_{Disc} = \frac{1}{2}MR_{Disc}^2 = I \text{ (given)}$$

$\therefore$ $M(R_{Disc})^2 = 2I$

Moment of inertia of sphere $I_{Sphere} = \frac{2}{5}MR_{Sphere}^2$

$$= \frac{2}{5}M\left(\frac{R_{Disc}}{2}\right)^2 = \frac{M}{10}(R_{Disc})^2 = \frac{2I}{10} = \frac{I}{5}$$

14. (3) N VD = (N – 1) MD

$1\ VD = \left(\frac{N-1}{N}\right)MD$

L.C. = Least count = 1MD – 1VD

$$L.C. = \left(1 - \frac{(N-1)}{N}\right)MD$$

$$= \frac{1}{N} \text{M.D.} = \frac{0.1}{N} \text{cm} = \frac{1}{10\,N} \text{cm}$$

$$= \frac{\text{value of 1 part on main scale}}{\text{number of parts on vernier scale}}$$

where V.D. = vernier division, M.D. Main scale division.

15. (2) The Gravitational field due to a thin spherical shell of radius R at distance r.

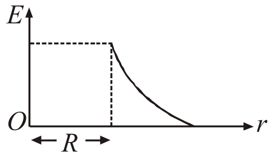
$E = \frac{GM}{r^2}$ (If $r > R$)

For $r = R$ i.e., on the surface of the shell

$E = \frac{GM}{R^2}$

For $r < R$ i.e., inside the shell

$E = 0$



16. (2) On a banked road,

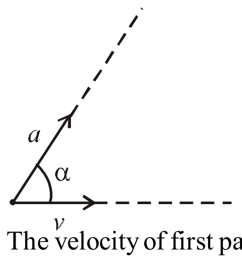
$$\frac{V_{max}^2}{Rg} = \left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)$$

Maximum safe velocity of a car on the banked road

$$V_{max} = \sqrt{Rg\left[\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right]}$$

17. (2) [Angular momentum]
= [moment of inertia] × [angular velocity]
= $[ML^2] \times [T^{-1}] = [ML^2T^{-1}]$

18. (2)



The velocity of first particle, $v_1 = v$
The velocity of second particle, $v_2 = at$
Relative velocity, $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$

or $v_{12}^2 = v^2 + (at)^2 - 2v(at\cos\alpha)$

For least value of relative velocity, $\frac{dv_{12}}{dt} = 0$

or $\frac{d}{dt}\left[v^2 + a^2t^2 - 2vat\cos\alpha\right] = 0$

or $0 + a^2 \times 2t - 2va\cos\alpha = 0$

or $t = \frac{v\cos\alpha}{a}$

19. (2) By energy conservation

$(K.E)_i + (P.E)_i = (K.E)_f + (P.E)_f$

$(K.E)_i = 0, (P.E)_i = mgh, (P.E)_f = 0$

$(K.E)_f = ½I\omega^2 + ½mv_{cm}^2$

Where I (moment of inertia) = $½mR^2$
(for solid cylinder)

so $mgh = ½(½mR^2)\left(\frac{v_{cm}^2}{R^2}\right) + ½mv_{cm}^2$

$\Rightarrow v_{cm} = \sqrt{4gh/3}$

20. (4) $K.E. = \frac{1}{2}mv^2$

Further, $v^2 = u^2 + 2as = 0 + 2ad = 2ad$
$= 2(F/m)d$

Hence, $K.E. = \frac{1}{2}m \times 2(F/m)d = Fd$

or, K.E. acquired = Work done
= F × d = constant.
i.e., it is independent of mass m.

21. (4) $\frac{mv^2}{(R+x)} = \frac{GmM}{(R+x)^2}$ also $g = \frac{GM}{R^2}$

$\therefore \frac{mv^2}{(R+x)} = m\left(\frac{GM}{R^2}\right)\frac{R^2}{(R+x)^2}$

$\therefore \frac{mv^2}{(R+x)} = mg\frac{R^2}{(R+x)^2}$

$\therefore v^2 = \frac{gR^2}{R+x} \Rightarrow v = \left(\frac{gR^2}{R+x}\right)^{1/2}$

22. (4) **Given :** Speed, $V = 54\ kmh^{-1} = 15\ ms^{-1}$
Moment of inertia, $I = 3\ kgm^2$
Time, t = 15s

$\omega_i = \frac{V}{r} = \frac{15}{0.45} = \frac{100}{3}$ $\omega_f = 0$

$\omega_f = \omega_i + \alpha t$

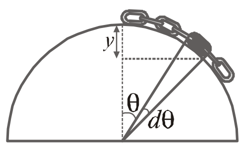
$0 = \frac{100}{3} + (-\alpha)(15)$ $\Rightarrow \alpha = \frac{100}{45}$

Average torque transmitted by brakes to the wheel

$\tau = (I)(\alpha) = 3 \times \frac{100}{45} = 6.66\ kgm^2s^{-2}$

23. (4) $dU = (dm)g(-y)$

$= -\frac{m}{\ell}(Rd\theta)gR(1-\cos\theta)$



$$= -\frac{mgR^2}{\ell}(1-\cos\theta)\,d\theta$$

$$\therefore \quad U = \int_0^{\theta=\frac{\ell}{R}} dU = -\frac{mgR^2}{\ell}\int_0^{\ell/R}(1-\cos\theta)d\theta$$

$$= -\frac{mgR^2}{\ell}(\theta-\sin\theta)_0^{\ell/R}$$

$$= \frac{mgR^2}{\ell}\left(\sin\frac{\ell}{R}-\frac{\ell}{R}\right).$$

24. (3) **Case-1 :** $u = 50\times\frac{5}{18}\text{m/s},$

$v = 0, s = 6\text{m}, a = a$

$v^2 - u^2 = 2as$

$$\Rightarrow \quad 0^2 - \left(50\times\frac{5}{18}\right)^2 = 2\times a\times 6$$

$$\Rightarrow \quad -\left(50\times\frac{5}{18}\right)^2 = 2\times a\times 6 \qquad \text{....(i)}$$

Case-2 : $u = 100\text{ km/hr} = 100\times\frac{5}{18}\text{ m/sec}$

$v = 0, s = s, a = a \qquad \therefore\ v^2 - u^2 = 2as$

$$\Rightarrow \quad 0^2 - \left(100\times\frac{5}{18}\right)^2 = 2as$$

$$\Rightarrow \quad -\left(100\times\frac{5}{18}\right)^2 = 2as \qquad \text{... (ii)}$$

Dividing (i) and (ii) we get

$$\frac{100\times100}{50\times50} = \frac{2\times a\times s}{2\times a\times 6} \Rightarrow s = 24\text{m}$$

25. (2) The orbital speed of satellite is independent of mass of satellite, so the ball will behave as a satellite and will continue to move with the same speed in the original orbit.

26. (2) We know that $F\times v = \text{Power}$

$\therefore\ F\times v = c$ where c = constant

$$\therefore\ m\frac{dv}{dt}\times v = c \qquad \left(\because F = ma = \frac{mdv}{dt}\right)$$

$$\therefore\ m\int_0^v vdv = c\int_0^t dt \qquad \therefore\ \frac{1}{2}mv^2 = ct$$

$$\therefore\ v = \sqrt{\frac{2c}{m}}\times t^{1/2}$$

$$\therefore\ \frac{dx}{dt} = \sqrt{\frac{2c}{m}}\times t^{1/2} \qquad \text{where } v = \frac{dx}{dt}$$

$$\therefore\ \int_0^x dx = \sqrt{\frac{2c}{m}}\times\int_0^t t^{1/2}dt$$

$$x = \sqrt{\frac{2c}{m}}\times\frac{2t^{3/2}}{3} \Rightarrow x \propto t^{3/2}$$

27. (3) $mg\sin\theta = ma$

$\therefore\ a = g\sin\theta$

where a is along the inclined plane

$\therefore$ vertical component of acceleration is $g\sin^2\theta$

$\therefore$ relative vertical acceleration of A with respect to B is

$$g(\sin^2 60 - \sin^2 30] = \frac{g}{2} = 4.9\ \text{m/s}^2$$

in vertical direction

28. (2) $\text{L.C.} = \frac{1}{100}\text{ mm}$

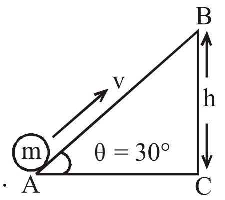
Diameter of wire = MSR + CSR × L.C.

$$= 0 + \frac{1}{100}\times 52$$

$$= 0.52\text{ mm} = 0.052\text{ cm}$$

29. (3) If a body rolls on a horizontal surface, it possesses both translational and rotational kinetic energies. The net kinetic energy is given by

$$K_{net} = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right),$$



where K is the radius of gyration.

So from law of conservation of energy,

$$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mgh,$$

where h is the height attained by the sphere.

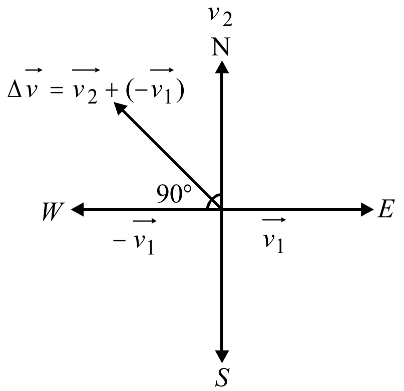
$$\text{i.e., } \frac{1}{2}\times2\times(10)^2\left(1+\frac{2}{5}\right) = 2\times9.8\times h.$$

$$\text{i.e., } \frac{1}{2}\times100\times\left(\frac{7}{5}\right) = 9.8h$$

$$\text{or} \quad h = \frac{700}{98} = 7.1\text{m}$$

30. (3) Average acceleration

$$= \frac{\text{change in velocity}}{\text{time interval}} = \frac{\Delta\vec{v}}{t}$$



$\vec{v_1} = 5\hat{i}, \vec{v_2} = 5\hat{j}$

$\Delta \vec{v} = (\vec{v}_2 - \vec{v}_1)$

$= \sqrt{v_1^2 + v_2^2 + 2v_1v_2 \cos 90}$

$= \sqrt{5^2 + 5^2 + 0}$

[As $|v_1| = |v_2| = 5$ m/s]

$= 5\sqrt{2}$ m/s

Avg. acc. $= \frac{\Delta \vec{v}}{t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s^2

$\tan\theta = \frac{5}{-5} = -1$

which means θ is in the second quadrant

i.e; (towards north-west)

PART B – CHEMISTRY

31. (1) For Balmer series, $n_1 = 2$ and $n_2 = 3, 4, 5$. For third line $n_1 = 2$ & $n_2 = 5$.

32. (3) $ICl_2^- \Rightarrow 2\ bp + 3lp$ (thus, sp^3d hybridisation) = linear

$BrF_2^+ \Rightarrow 2\ bp + 2lp$ (thus, sp^3 hybridisation) = pyramidal

$ClF_4^- \Rightarrow 4\ bp + 2lp$ (thus, sp^3d^2 hybridisation) = square planar

$AlCl_4^- \Rightarrow 4\ bp + 0lp$ (thus sp^3 hybridisation) = tetrahedral

33. (4) $\frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3} = 1.67$

34. (1) Normality of ferrous Amm. sulphate

$= \frac{3.92 \times 1000}{392\,(\text{eq. wt}) \times 100} = 0.1$

$N_1V_1 = N_2V_2$

$20 \times 0.1 = 18 \times N_2$

$N_2 = 0.111$

1 gev. of $KMnO_4 = 31.6$ g

0.111 gev. of $KMnO_4 = 31.6 \times 0.111 = 3.5$ g.

35. (1) Dissolution of NaOH in water is an exothermic reaction as indicated by negative value of enthalpy leading to increase in temperature.

36. (3) Mass of 1 L of vapour = volume × density $= 1000 \times 0.0006 = 0.6$ g

Volume of liquid water $= \frac{\text{mass}}{\text{density}} = \frac{0.6}{1} = 0.6\ \text{cm}^3$

37. (1) Both XeF_2 and CO_2 have a linear structure.

F — Xe — F   O = C = O

38. (3) Reaction (iii) can be obtained by adding reactions (i) and (ii) therefore $K_3 = K_1 . K_2$

Hence (iii) is the correct answer.

39. (3) According to de Broglie,

$\lambda = \frac{h}{mv} \Rightarrow mv = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{10^{-17}}$

$\Rightarrow p = 6.626 \times 10^{-17}$ kg m/s

40. (3) $K_p = K_c (RT)^{\Delta n}$;

Here $\Delta n = 1 - 2 = -1$

$\therefore \frac{K_p}{K_c} = \frac{1}{RT}$

41. (4) K.E. of the molecule per atom

$= \frac{(4.4 \times 10^{-19}) - (4.0 \times 10^{-19})}{2}$

$= \frac{0.4 \times 10^{-19}}{2} = 2.0 \times 10^{-20}$ J

42. (3) Both XeF_2 and IF_2^- are sp^3d hybridized and have trigonal bipyramidal (linear) shape due to presence of 3 *lp* of electrons.

43. (2) $X_2 + Y_2 \longrightarrow 2XY, \Delta H = 2(-200)$.

Let x be the bond dissociation energy of X_2. Then

$\Delta H = -400 = B.E_{x-x} + B.E_{y-y} - 2B.E_{x-y}$

$= x + 0.5x - 2x = -0.5x$

or $x = \frac{400}{0.5} = 800\ \text{kJ mol}^{-1}$

44. (3) Given $p_A = 1$ atm

$p_A + p_B = 2$ atm

$\therefore p_B = 1$ atm

From ideal gas equation, pV = nRT

$p \propto n$ (V, R and T are constant)

$\frac{W_A}{M_A} = \frac{W_B}{M_B}$

$\frac{M_A}{M_B} = \frac{W_A}{W_B} = \frac{4}{6} = \frac{2}{3}$

45. (1) The dipole moment of two dipoles inclined at an angle θ is given by the equation :

$\mu = \sqrt{X^2 + Y^2 + 2XY\cos\theta}$, $\cos 90° = 0$.

Since the angle increases from 90° to 180°, the value of cos θ becomes more and more – *ve* and hence resultant dipole moment decreases. Thus, dipole moment is maximum when θ = 90°.

46. (2) Total atoms in 1 molecule of $C_{12}H_{22}O_{11}$ $= 12 + 22 + 11 = 45$

∴ Total atoms in 1 mole of $C_{12}H_{22}O_{11}$ $= 45 \times 6.02 \times 10^{23}$ atoms/mol.

47. (1) $CaCO_3 \longrightarrow \underset{x}{Ca^{2+}} + \underset{x}{CO_3^{2-}}$

$CaC_2O_4 \longrightarrow \underset{y}{Ca^{2+}} + \underset{y}{C_2O_4^{2-}}$

$\therefore [Ca^{2+}] = x + y$

Now, $K_{sp}(CaCO_3) = [Ca^{2+}][CO_3^{2-}]$

or $4.7 \times 10^{-9} = (x + y)x$

similarly, $K_{sp}(CaC_2O_4) = [Ca^{2+}][C_2O_4^{2-}]$
or $1.3 \times 10^{-9} = (x+y)y$
On solving, we get
$[Ca^{2+}] = 7.746 \times 10^{-5}$ M

48. (2) $6 = -\log 10^{-5} + \log \frac{SALT}{ACID} = 5 + \log \frac{SALT}{ACID}$

$\log \frac{SALT}{ACID}$ must be 1. $\therefore \frac{SALT}{ACID} = \frac{10}{1}$ or 10 : 1 .

49. (2) According to Fajan's rule

Covalent character : $\overset{4+}{CCl_4} > \overset{3+}{BCl_3} > \overset{2+}{BeCl_2} > \overset{+1}{LiCl}$

50. (4) $Z = \frac{P_C V_C}{RT_C}$; also, gaseous molecules showing H-bonding show maximum deviations in Z due to increase in molecular attractions (e.g., Z for NH_3, H_2O, $CH_3OH \simeq 0.22$ to 0.24

51. (1) Hybridisation = $\frac{1}{2}$ [No. of valence electrons of central atom + No. of monovalent atoms attached to it + Negative charge if any – Positive charge if any]

NO_2^-, $H = \frac{1}{2}[5+0+1-0] = 3 = sp^2$

NO_3^-, $H = \frac{1}{2}[5+0+1-0] = 3 = sp^2$

NH_2^-, $H = \frac{1}{2}[5+2+1+0] = 4 = sp^3$

NH_4^+, $H = \frac{1}{2}[5+4+0-1] = 4 = sp^3$

$SCN^- = sp$

NO_2^- and NO_3^- have same hybridisation.

52. (2) As the size of alkali metal ion increases, lattice enthalpy decreases and hence the stability of the corresponding metal hydride decreases.

53. (1) For $n = 5$, l may be 0, 1, 2, 3 or 4
For $l = 4$, $m = 2l + 1 = 2 \times 4 + 1 = 9$
$= -4, -3, -2, -1, 0, +1, +2, +3, +4$

For $m = 0$, $s = +\frac{1}{2}$

Hence, (1) is correct option.

(2) For any value of n, the value of l cannot be equal or greater than value of n, hence it is incorrect.

(3) For $l = 0$, $m = 0$ hence it is incorrect.

(4) The value of s can never be zero. Thus this option is also incorrect.

54. (1) For spontaneous reaction, dS > 0 and dG should be negative i.e. < 0.

55. (3)

Element	%	Relative no. of atoms	Simplest ratio of atoms
C	49.3	49.3/12 = 4.1	4.1/2.74 = 1.5 1.5 × 2 = 3
H	6.84	6.84/1 = 6.84	6.84/2.74 = 2.5 = 2.5 × 2 = 5
O	43.86	43.86/16 = 2.74	2.74/2.74 = 1 1 × 2 = 2

$\therefore$ Empirical formula = $C_3H_5O_2$

Empirical formula mass
$= (3 \times 12) + (5 \times 1) + (2 \times 16) = 36 + 5 + 32 = 73$

Molecular mass = 2 × Vapour density
$= 2 \times 73 = 146$

$n = \frac{\text{molecular mass}}{\text{empirical formula mass}} = 146/73 = 2$

Molecular formula = Empirical formula × 2
$= (C_3H_5O_2) \times 2 = C_6H_{10}O_4$

56. (2) We are given,

$C_6H_6(l) + \frac{15}{2}O_2(g) \rightarrow 6CO_2(g) + 3H_2O(l)$

$\Delta H = -3270 \text{ kJ mol}^{-1}$ ...(i)

$C_{graphite} + O_2(g) \rightarrow CO_2(g),$

$\Delta H = -394 \text{ kJ mol}^{-1}$ ...(ii)

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l),$

$\Delta H = -286 \text{ kJ mol}^{-1}$ ...(iii)

Formation of C_6H_6

$6C_{graphite} + 3H_2(g) \rightarrow C_6H_6(l); \Delta H = ?$ ...(iv)

By multiplying eq. (ii) with 6 and eq. (iii) with 3 and adding we get,

$6C(graphite) + 6O_2(g) + 3H_2(g) + \frac{3}{2}O_2(g)$

$\rightarrow 6CO_2(g) + 6H_2O(l)$

$\Delta H = 6(-394) + 3(-286) = (-2364) + (-858)$
$= -3222$ kJ/ mol

Now, by substracting eq. (i) from (v) we get

$6C_{graphite} + 3H_2(g) \rightarrow C_6H_6(l)$

$\Delta H = -3222 - (-3270) = +48 \text{ kJ / mol}$

57. (4) Calculating the bond order of various species.

$O_2^- : KK\,\sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2,$

$\pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^1$

$B.O. = \frac{1}{2}(N_b - N_a)$

$= \frac{8-5}{2}$ or 1.5

$NO : KK\,\sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^1$

$B.O. = \frac{N_b - N_a}{2} = \frac{8-3}{2}$ or 2.5

$C_2^{2-} : KK\,\sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2$

$B.O. = \frac{N_b - N_a}{2} = \frac{8-2}{2}$ or 3

$He_2^{2+} = \sigma 1s^2 \sigma^* 1s^1$

$B.O. = \frac{N_b - N_a}{2} = \frac{2-1}{2}$ or 0.5

From these values we conclude that the correct order of increasing bond order is

$He_2^{2+} < O_2^- < NO < C_2^{2-}$

58. (2)

	H_2	+	I_2	$\rightleftharpoons$	2HI
Initial	1		1		0
Final	$\frac{1-\alpha}{v}$		$\frac{1-\alpha}{v}$		$\frac{2\alpha}{v}$

Since $\Delta n = [2-(1+1)] = 0$

$\therefore$ K does not depend on volume.

So on reducing volume to half K does not change.

$\therefore$ K = 50.

59. (4)

HCl	+	NaOH	$\longrightarrow$	NaCl	+	H_2O
75×0.2		25×0.2				
= 15		5				
10		0		5		5

$\left[H^+\right] = \frac{10}{100} = 0.1$

pH = 1

60. (3) Enthalpy of neutralisation of HCl with NaOH is x, in question geq of HCl

$\left(\frac{500\times2}{1000} = 1\text{geq}\right)$ and NaOH $\left(\frac{250\times4}{1000} = 1\text{ geq}\right)$

Hence the value heat evolved is x.

PART C – MATHEMATICS

61. (2) Given set can be written as
(A – B) ∪ (B – A) = (A ∪ B) – (A ∩ B)
(By definition of symmetric difference)
Hence, (A \ B) ∪ (B \ A) = (A ∪ B) \ (A ∩ B)

62. (1) $f(x)$ is defined if $-\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) - 1 > 0$

$\Rightarrow \log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) < -1$

$\Rightarrow 1+\frac{1}{x^{1/4}} > \left(\frac{1}{2}\right)^{-1}$

$\Rightarrow \frac{1}{x^{1/4}} > 1$

$\Rightarrow 0 < x < 1$

63. (3) Here A and B sets having 2 elements in common, so A × B and B × A have 2^2 i.e., 4 elements in common. Hence, $n[(A\times B)\cap(B\times A)] = 4$

64. (1) $|x_1z_1 - y_1z_2|^2 + |y_1z_1 + x_1z_2|^2$

$= |x_1z_1|^2 + |y_1z_2|^2 - 2\text{Re}(x_1y_1z_1\bar{z}_2)$
$+ |y_1z_1|^2 + |x_1z_2|^2 + 2\text{Re}(x_1y_1z_1\bar{z}_2)$

$= x_1^2|z_1|^2 + y_1^2|z_2|^2 + y_1^2|z_1|^2 + x_1^2|z_2|^2$

$= 2(x_1^2 + y_1^2)(4^2) = 32(x_1^2 + y_1^2)$

65. (1) Given, $\sin\theta + \text{cosec}\,\theta = 2$ ....(i)

Squaring on both side of equation (i)

$(\sin\theta + \text{cosec}\,\theta)^2 = 2^2$

$\Rightarrow \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\frac{1}{\sin\theta} = 4$

$\left(\because \text{cosec}\,\theta = \frac{1}{\sin\theta}\right)$

$\sin^2\theta + \text{cosec}^2\theta = 2$ .... (ii)

Cubing on both side of equation (ii)

$(\sin^2\theta + \text{cosec}^2\theta)^3 = 2^3$

$\Rightarrow \sin^6\theta + \text{cosec}^6\theta + 3\sin^2\theta\,\text{cosec}^2\theta$
$(\sin^2\theta + \text{cosec}^2\theta) = 8$

$\Rightarrow \sin^6\theta + \text{cosec}^6\theta + 3\sin^2\theta\frac{1}{\sin^2\theta}\times 2 = 8$ [from(ii)]

$\sin^6\theta + \text{cosec}^6\theta = 2$ .... (iii)

Again squaring on both side of equation (ii)

$(\sin^2\theta + \text{cosec}^2\theta)^2 = 2^2$

$\Rightarrow \sin^4\theta + \text{cosec}^4\theta + 2\sin^2\theta\,\text{cosec}^2\theta = 4$

$\Rightarrow \sin^4\theta + \text{cosec}^4\theta + 2\sin^2\theta\frac{1}{\sin^2\theta} = 4$

$\Rightarrow \sin^4\theta + \text{cosec}^4\theta = 2$ .... (iv)

From equation (iii) × equation (iv), we get

$(\sin^6\theta + \text{cosec}^6\theta)(\sin^4\theta + \text{cosec}^4\theta) = 2\times 2$

$\Rightarrow \sin^6\theta\sin^4\theta + \sin^6\theta\,\text{cosec}^4\theta +$
$\text{cosec}^6\theta\sin^4\theta + \text{cosec}^6\theta.\text{cosec}^4\theta = 4$

$\Rightarrow \sin^{10}\theta + \sin^2\theta + \text{cosec}^2\theta + \text{cosec}^{10}\theta = 4$

$\Rightarrow \sin^{10}\theta + \text{cosec}^{10}\theta + (\sin^2\theta + \text{cosec}^2\theta) = 4$

$\Rightarrow \sin^{10}\theta + \text{cosec}^{10}\theta + 2 = 4$ [from equation (ii)]

$\Rightarrow \sin^{10}\theta + \text{cosec}^{10}\theta = 2$

66. (3) Given that,

$f(x) = \frac{1}{2} - \tan\left(\frac{\pi x}{2}\right), -1 < x < 1$

Here, domain of $f(x) = (-1, 1)$ and

$g(x) = \sqrt{3+4x-4x^2} = \sqrt{-(2x-3)(2x+1)}$

$\Rightarrow (2x-3)(2x+1) \le 0$

$\Rightarrow -\frac{1}{2} \le x \le \frac{3}{2}$

$\therefore$ Domain of $g(x) = \left[-\frac{1}{2}, \frac{3}{2}\right]$

Hence, domain of $(f+g)$ = intersection of their domains

$= \left[-\frac{1}{2}, 1\right)$.

67. (2) We have

$$|z| = \left|z + \frac{2}{z} - \frac{2}{z}\right| \le \left|z + \frac{2}{z}\right| + \frac{2}{|z|}$$

$[\because \; |z_1 \pm z_2| \le |z_1| + |z_2|]$

$\Rightarrow \left(3^{\frac{1}{2}} + 2^{\frac{1}{2}}\right)$

$\Rightarrow |z|^2 - 2|z| + 1 \le 1 + 2 \Rightarrow (|z| - 1)^2 \le 3$

$\Rightarrow -\sqrt{3} \le |z| - 1 \le \sqrt{3}$

$\Rightarrow 1 - \sqrt{3} \le |z| \le 1 + \sqrt{3}$

Thus, the maximum value of $|z|$ is $1 + \sqrt{3}$

68. (1) ${}^{x+2}P_{x+2} = a$

$\Rightarrow a = (x+2)!$

${}^{x}P_{11} = b$

$\Rightarrow b = \dfrac{x!}{(x-11)!}$ and ${}^{x-11}P_{x-11} = c$

$\Rightarrow c = (x-11)!$

$a = 182\, bc$

$\therefore (x+2)! = 182 \dfrac{x!}{(x-11)!} (x-11)!$

$\Rightarrow (x+2)(x+1) = 182 = 14 \times 13$

$\therefore x + 1 = 13$

$\therefore x = 12$

69. (4) We write the numbers in the product (n!) ! in rows containing n numbers each. So, (n !) ! =

$1 . 2 . 3 . 4 \ldots\ldots\ldots\ldots n$
$(n+1)(n+2)(n+3) \ldots\ldots\ldots 2n$
$(2n+1)(2n+2)(2n+3) \ldots\ldots\ldots\ldots 3n$
$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
$[\{(n-1)!-1\} n+1]\, [\{(n-1)!-1\} n+2]$

} Total (n–1) ! rows

$\times [\{(n-1)!-1\} n+3] \ldots\ldots n!$

Now each row is divisible by n!

So, (n!)! is divisible by n! × n! ×........(n–1)! times $= (n!)^{(n-1)!}$

70. (2) Let $u = \cos\theta\left\{\sin\theta + \sqrt{\sin^2\theta + \sin^2\alpha}\right\}$

$\Rightarrow (u - \sin\theta\cos\theta)^2 = \cos^2\theta\,(\sin^2\theta + \sin^2\alpha)$

$\Rightarrow u^2\tan^2\theta - 2u\tan\theta + u^2 - \sin^2\alpha = 0$

Since $\tan\theta$ is real, therefore

$\Rightarrow 4u^2 - 4u^2(u^2 - \sin^2\alpha) \ge 0$

$\Rightarrow u^2 - (1 + \sin^2\alpha) \le 0 \Rightarrow |u| \le \sqrt{1 + \sin^2\alpha}.$

71. (1) We know that,

$(a-1)^n = {}^nC_0.a^n - {}^nC_1.a^{n-1} + {}^nC_2a^{n-2} - \ldots\ldots\ldots + (-1)^{n-1}.{}^nC_{n-1}a + (-1)^n\,{}^nC_n$

$\therefore \dfrac{(a-1)^n}{a} = {}^nC_0a^{n-1} - {}^nC_1a^{n-2} + {}^nC_2a^{n-3} - \ldots\ldots\ldots + (-1)^{n-1}.{}^nC_{n-1} + \dfrac{(-1)^n}{a}\,{}^nC_n$

$\therefore f(n) = \dfrac{(a-1)^n - (-1)^n}{a}$

$\therefore f(2007) + f(2008) = \dfrac{(a-1)^{2007} + 1}{a} + \dfrac{(a-1)^{2008} - 1}{a}$

$= \dfrac{(a-1)^{2007}(1 + a - 1)}{a} = (a-1)^{2007}$

$= \left(3^{\frac{1}{223}}\right)^{2007} = 3^9 = 3^2 \cdot 3^7 = 9\,(2187)$

$\therefore k = 2187$

72. (4) T_n denotes the number of triangles which can be formed by using the vertices of a regular polygon of n sides.

$\therefore T_n = {}^nC_3$

$\Rightarrow T_{n+1} = {}^{n+1}C_3$

$\Rightarrow T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 28$ (Given)

$\dfrac{(n+1)!}{3!(n+1-3)!} - \dfrac{n!}{3!(n-3)!} = 28$

$\Rightarrow \dfrac{(n+1)n(n-1)}{6} - \dfrac{n(n-1)(n-2)}{6} = 28$

$\Rightarrow \dfrac{n(n-1)}{6} \times 3 = 28$

$\Rightarrow n(n-1) = 28 \times 2$

$\Rightarrow (n-8)(n+7) = 0$

$\Rightarrow n = 8, -7$

n can never be less than zero

$\Rightarrow n = 8$

73. (3) Given, $\tan A = \dfrac{1}{2}$, $\tan B = \dfrac{1}{3}$ … (i)

Now, $\tan(2A + B)$

$$= \frac{\tan 2A + \tan B}{1 - \tan 2A \tan B} = \frac{\dfrac{2\tan A}{1 - \tan^2 A} + \tan B}{1 - \dfrac{2\tan A}{1 - \tan^2 A} \times \tan B}$$

$$= \frac{\left(\dfrac{2 \times \frac{1}{2}}{1 - \frac{1}{4}}\right) + \dfrac{1}{3}}{1 - \left(\dfrac{2 \times \frac{1}{2}}{1 - \frac{1}{4}}\right) \times \dfrac{1}{3}} = \frac{\frac{4}{3} + \frac{1}{3}}{1 - \frac{4}{3} \times \frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{9}} = 3.$$

74. **(1)** $|x_1z_1 - y_1z_2|^2 + |y_1z_1 - x_1z_2|^2$

$= |x_1z_1|^2 + |y_1z_2|^2 - 2\text{Re}(x_1y_1z_1z_2)$

$+ |y_1z_1|^2 + |x_1z_2|^2 + 2\text{Re}(x_1y_1z_1z_2)$

$= x_1^2|z_1|^2 + y_1^2|z_2|^2 + y_1^2|z_1|^2 + x_1^2|z_2|^2$

$= x_1^2|z_1|^2 + y_1^2|z_2|^2 + y_1^2|z_1|^2 + x_1^2|z_2|^2$

$= 2(x_1^2 + y_1^2)(4^2) = 32(x_1^2 + y_1^2)$

75. **(2)** Since α, β are the roots of the equation $2x^2 - 35x + 2 = 0$

$\therefore$ $\alpha\beta = 1$

$\therefore$ $2\alpha^2 - 35\alpha = -2$ or $2\alpha - 35 = \dfrac{-2}{\alpha}$

$2\beta^2 - 35\beta = -2$ or $2\beta - 35 = \dfrac{-2}{\beta}$

Now, $(2\alpha - 35)^3 (2\beta - 35)^3 = \left(\dfrac{-2}{\alpha}\right)^3 \left(\dfrac{-2}{\beta}\right)^3$

$= \dfrac{8\times 8}{\alpha^3\beta^3} = \dfrac{64}{1} = 64.$

76. **(1)** Given $f(x) = \cos(\log x)$

$\therefore$ $f(xy) = \cos(\log xy)$

$f(xy) = \cos[\log x + \log y]$ ....(i)

And $f\left(\dfrac{x}{y}\right) = \cos\left(\log\dfrac{x}{y}\right)$

$f\left(\dfrac{x}{y}\right) = \cos(\log x - \log y)$ ....(ii)

Adding (i) and (ii), we get

$f(xy) + f\left(\dfrac{x}{y}\right) = \cos(\log x + \log y) + \cos(\log x - \log y)$

$= 2\cos(\log x).\cos(\log y)$

$\Rightarrow$ $f(xy) + f\left(\dfrac{x}{y}\right) = 2f(x).f(y)$

Then the value of $f(x)f(y) - \dfrac{1}{2}\left\{f\left(\dfrac{x}{y}\right) + f(xy)\right\}$

$= f(x)\,f(y) - \dfrac{1}{2}.2\{f(x).f(y)\} = 0$

77. **(2)** The last term $= {}^nC_n.\left(-\dfrac{1}{\sqrt{2}}\right)^n = \left(\dfrac{1}{3.\sqrt[3]{9}}\right)^{\log_3 8}$

(from the question)

$(-1)^n.\left(\dfrac{1}{2}\right)^{n/2} = \left(\dfrac{1}{3^{5/3}}\right)^{\log_3 8}$

$= 3^{(-5/3).3\log_3 2} = 3^{\log_3 2^{-5}} = 2^{-5} = \left(\dfrac{1}{2}\right)^5$

$\Rightarrow$ $n = 10$

So, $t_5 = {}^{10}C_4.(2^{1/3})^6.\left(-\dfrac{1}{\sqrt{2}}\right)^4$

$= {}^{10}C_4 = {}^{10}C_{10-4} = {}^{10}C_6$

78. **(2)** Let

$S = 2\log(x+h) - \log_e x - \left\{\dfrac{h^2}{(x+h)^2} + \dfrac{1}{2}\dfrac{h^4}{(x+h)^4} + \dfrac{1}{3}\dfrac{h^6}{(x+h)^6} + ...\right\}$

Put $\dfrac{h^2}{(x+h)^2} = y$, then we have

$S = 2\log_e(x+h) - \log_e x - \left\{y + \dfrac{y^2}{2} + \dfrac{y^3}{3} + ...\right\}$

$= 2\log_e(x+h) - \log_e x + \log_e(1-y)$

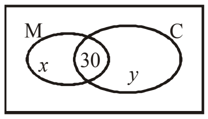
$= 2\log_e(x+h) - \log_e x + \log_e\left\{1 - \dfrac{h^2}{(x+h)^2}\right\}$

$= \log_e(x+h)^2 - \log_e x + \log_e\left\{\dfrac{x^2+2hx}{(x+h)^2}\right\}$ \

$= \log_e(x+h)^2 - \log_e x + \log_e(x^2+2hx) - \log_e(x+h)^2$

$= \log_e\left(\dfrac{x^2+2hx}{x}\right) = \log_e(x+2h)$

79. **(1)** Let the number of students who take only mathematics be x and only chemistry be y.



So, from the Venn diagram, we have total number of students who take mathematics $= x + 30$ and take chemistry $= y + 30$.

According to the question, we have

$30 = \dfrac{10}{100}(x + 30)$

$\Rightarrow$ $x = 270$ and

$30 = \dfrac{12}{100}(30 + y)$

$\Rightarrow$ $y = 220$

$x + y + 30 = 270 + 220 + 30 = 520.$

80. **(1)** We have $\cos 3x + \sin\left(2x - \dfrac{7\pi}{6}\right) = -2$

$\Rightarrow$ $1 + \cos 3x + 1 + \sin\left(2x - \dfrac{7\pi}{6}\right) = 0$

$\Rightarrow$ $(1 + \cos 3x) + 1 - \cos\left(2x - \dfrac{2\pi}{3}\right) = 0$

$\Rightarrow$ $2\cos^2\dfrac{3x}{2} + 2\sin^2\left(x - \dfrac{\pi}{3}\right) = 0$

$\Rightarrow$ $\cos\dfrac{3x}{2} = 0$ and $\sin\left(x - \dfrac{\pi}{3}\right) = 0$

$\Rightarrow \quad \frac{3x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, ...$ and $x - \frac{\pi}{3} = 0, \pi, 2\pi........$

$\Rightarrow \quad x = \frac{\pi}{3}$

Therefore, the general solution of $\cos\frac{3x}{2} = 0$ and

$\sin\left(x - \frac{\pi}{3}\right) = 0$ is $x = 2k\pi + \frac{\pi}{3} = \frac{\pi}{3}(6k+1)$,

where $k \in Z$.

81. **(4)** $z + |z| = 8 + 12i$

$\Rightarrow \quad x + iy + \sqrt{x^2 + y^2} = 8 + 12i$

$\Rightarrow \quad x + \sqrt{x^2 + y^2} = 8$ and $y = 12$

So, $z = -5 + 12i$

$\Rightarrow \quad |z| = \sqrt{25 + 144} = 13$

$\Rightarrow \quad |z^2| = |z|^2 = 169$

82. **(2)** We know that $\frac{C_k}{C_{k-1}} = \frac{{}^nC_k}{{}^nC_{k-1}} = \frac{n-k+1}{k}$

$\therefore \quad \sum_{k=1}^{n} k^3\left(\frac{C_k}{C_{k-1}}\right)^2 = \sum_{k=1}^{n} k^3\left(\frac{n-k+1}{k}\right)^2 = \sum_{k=1}^{n} k(n-k+1)^2$

Put $n - k + 1 = p \Rightarrow k = n - p + 1$

when $k = 1$, then $p = n$ and when $k = n$, $p = 1$.

$\therefore \quad$ Series $= \sum_{p=n}^{1}(n-p+1)p^2 = \sum_{p=1}^{n}(np^2 - p^3 + p^2)$

$= \sum_{p=1}^{n}(n+1)p^2 - \sum_{p=1}^{n}p^3$

$= \frac{(n+1)n(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4}$

$= \frac{n(n+1)^2}{2}\left[\frac{2n+1}{3} - \frac{n}{2}\right] = \frac{n(n+1)^2(n+2)}{12}$

83. **(4)** Triangles with vertices on AB, BC and CD are

$3 \times 4 \times 5 = 60$

Triangles with vertices on AB, BC and DA are

$3 \times 4 \times 6 = 72$

Triangles with vertices on AB, CD and DA are

$3 \times 5 \times 6 = 90$

Triangles with vertices on BC, CD and DA are

$4 \times 5 \times 6 = 120$

$\therefore \quad$ Total no. of triangles $= 60 + 72 + 90 + 120 = 342$

84. **(4)** (1) Expression on expansion gives

$1 + 1000 \times 10^{-4} + \frac{1000 \times 999}{2}10^{-8} + {}^{1000}C_3 10^{-12} + ...$

$< 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + ... = \frac{1}{1 - \frac{1}{10}} = \frac{10}{9}$

So, integer just greater than the given expression must be 2.

(2) In the expansion of $\left(7^{\frac{1}{3}} + 11^{\frac{1}{9}}\right)^{6561}$

$t_{r+1} = {}^{6561}C_r 7^{\frac{1}{3}(6561-r)} 11^{\frac{r}{9}} = {}^{6561}C_r 7^{\left(2187 - \frac{r}{3}\right)} 11^{\frac{r}{9}}$

But t_{r+1} will be an integer if both $\frac{r}{3}, \frac{r}{9}$ are integers and so r must be a multiple of 9 and $0 \le r \le 6561$. Let such numbers be n. Then value of such r must form an A.P. with the first term = 0 and common difference = 9. Hence $0 + (n-1)9 = 6561$

or $n - 1 = 729$ *or* $n = 729 + 1$ or $n = 730$.

Hence, number of integral terms = 730.

(3) $\because \quad \left(\sqrt[4]{9} + \sqrt[6]{8}\right)^{500} = \left(3^{\frac{1}{2}} + 2^{\frac{1}{2}}\right)^{500}$

$\therefore \quad$ General term,

$T_{r+1} = {}^{500}C_r 3^{\frac{500-r}{2}} 2^{\frac{r}{2}} = {}^{500}C_r 3^{250} 3^{\frac{-r}{2}} 2^{\frac{r}{2}}$

For integral terms r is multiple of 2

Let $r = 2\lambda$, $\lambda \in \mathbf{I}$ and $0 \le r \le 500$

$\therefore \quad r = 0, 2, 4, 6, 8, 10, ..., 500$

$\therefore \quad$ No. of integral terms $= 1 + 250 = 251$

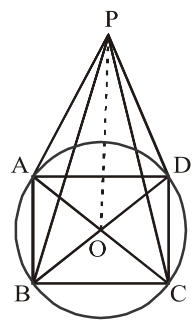
85. **(3)** OP = 4 cm

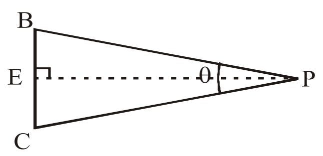
OA = OB = OC = OD = 3 cm

$AP = BP = CP = DP = \sqrt{3^2 + 4^2} = 5$ cm

ABCD is a square of side length

$= \frac{6}{\sqrt{2}} = 3\sqrt{2}$ cm



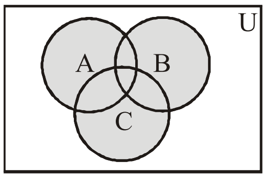


In ΔBEP

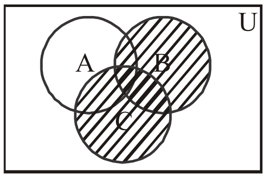
$$\sin\frac{\theta}{2} = \frac{\frac{1}{2}BC}{BP} = \frac{\frac{3}{\sqrt{2}}}{5} = \frac{3}{5\sqrt{2}}$$

$$\therefore \quad \cos\theta = 1 - 2\sin^2\frac{\theta}{2} = \frac{16}{25}$$

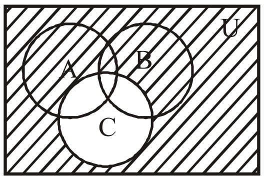
86. (1)



(i) $A \cup B \cup C$



(ii) $(A \cap B^c \cap C^c)^c$



(iii) C^c

From Fig. (i), (ii) and (iii), we get

$(A \cup B \cup C) \cap (A \cap B^c \cap C^c)^c \cap C^c = (B \cap C^c)$

Alternative Solution:

$(A \cup B \cup C) \cap (A \cap B^c \cap C^c)^c \cap C^c$

$(A \cup B \cup C) \cap (A^c \cup \, B^c)^c \cup (C^c)^c) \cap C^c$

[By Demorgan's Law]

$[(A \cup B \cup C) \cap (A^c \cup B \cup C)] \cap C^c$

[By Associative property]

$[(A \cap A^c) \cup (B \cup C)] \cap C^c$ [By Distributive property]

$(B \cup C) \cap C^c = (B \cap C^c) \cup (C \cap C^c) = B \cap C^c$

[$\because X \cap X^c = \phi$]

87. (4) Since the largest digit is in the middle, the middle digit is greater than or equal to 4, the number of numbers with 4 in the middle $= {}^4P_4 - {}^3P_3$.

($\because$ The other four places are to be filled by 0, 1, 2 and 3, and a number cannot begin with 0). Similarly, the numbers of numbers with 5 in the middle $= {}^5P_4 - {}^4P_3$, etc.)

$\therefore$ The required number of numbers

$$= ({}^4P_4 - {}^3P_3) + ({}^5P_4 - {}^4P_3) + ({}^6P_4 - {}^5P_3) + \ldots + ({}^9P_4 - {}^8P_3)$$

$$= \sum_{n=4}^{9} {}^nP_4 - \sum_{n=3}^{8} {}^nP_3$$

88. (2) If α is the smallest positive angle for which $\sin\alpha = x$, then $\beta = \pi - \alpha$, $\gamma = 2\pi + \alpha$ and $\delta = 3\pi - \alpha$

So, $4\sin\frac{\alpha}{2} + 3\sin\frac{\beta}{2} + 2\sin\frac{\gamma}{2} + \sin\frac{\delta}{2}$

$$= 4\sin\frac{\alpha}{2} + 3\cos\frac{\alpha}{2} - 2\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}$$

$$= 2\sin\frac{\alpha}{2} + 2\cos\frac{\alpha}{2} = 2\sqrt{1+\sin\alpha} = 2\sqrt{1+x}$$

89. (4) $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\sin\theta = (\sqrt{2}+1)\cos\theta$

$\Rightarrow \tan\theta = \sqrt{2}+1$

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\cos\theta = (\sqrt{2}-1)\sin\theta$

$\Rightarrow \tan\theta = \sqrt{2}+1$

$\therefore P = Q$

90. (4) We have, $a = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$

$$\Rightarrow a^7 = \left[\cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)\right]^7$$

$= \cos 2\pi + i\sin 2\pi = 1$ …(i)

Let $s = \alpha + \beta = (a + a^2 + a^4) + (a^3 + a^5 + a^6)$

[$\because \alpha = a + a^2 + a^4, \beta = a^3 + a^5 + a^6$]

$$\Rightarrow S = a + a^2 + a^3 + a^4 + a^5 + a^6 = \frac{a(1-a^6)}{1-a}$$

$$\Rightarrow S = \frac{a - a^7}{1-a} = \frac{a-1}{1-a} = -1 \quad \ldots\text{(ii)}$$

Let $P = \alpha\beta = (a + a^2 + a^4)(a^3 + a^5 + a^6)$

$= a^4 + a^6 + a^7 + a^5 + a^7 + a^8 + a^7 + a^9 + a^{10}$

$= a^4 + a^6 + 1 + a^5 + 1 + a + 1 + a^2 + a^3$

[from Eq. (i)]

$= 3 + (a + a^2 + a^3 + a^4 + a^5 + a^6) = 3 + S$

$= 3 - 1 = 2$ [from Eq. (ii)]

Required equation is, $x^2 - Sx + P = 0$

$\Rightarrow x^2 + x + 2 = 0$