JEE MAIN TEST SERIES

SOLUTIONS PART TEST-2

PART A – PHYSICS

1. **(3)** We know that

$$Y = \frac{mg/A}{\Delta\ell/\ell} = \frac{mg\ell}{A\Delta\ell} \quad ....(i)$$

Also $\Delta\ell = \ell\,\alpha\,\Delta T$ ...(ii)

From (1) and (2)

$$Y = \frac{mg\ell}{A\ell\,\alpha\,\Delta T} = \frac{mg}{A\,\alpha\,\Delta T}$$

$$\Rightarrow \quad m = \frac{YA\,\alpha\,\Delta T}{g} = \frac{10^{11}\times\pi(10^{-3})^2\times10^{-5}\times10}{10}$$

$\therefore \quad m \approx 3$

2. **(3)** Velocity of ball when it strikes the water surface

$v = \sqrt{2gh}$ ...(i)

Terminal velocity of ball inside the water

$$v = \frac{2}{9}r^2 g\frac{(\rho-1)}{\eta} \quad ...(ii)$$

From equation (i) and (ii) we get $\sqrt{2gh} = \frac{2}{9}\frac{r^2 g}{\eta}(\rho-1)$

$$\Rightarrow h = \frac{2}{81}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$$

3. **(2)** $dW = P\,\Delta\,V = 1.01\times10^5\,[1671-1]\times10^{-6}$ Joule

$$= \frac{1.01\times167}{4.2}\text{cal.}$$

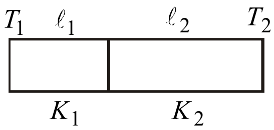
$= 40$ cal. nearly

$\Delta Q = mL = 1\times540,$

$\Delta Q = \Delta W + \Delta U$

or $\Delta U = 540 - 40 = 500$ cal.

4. **(4)** Let T be the temperature of the interface. As the two sections are in series, the rate of flow of heat in them will be equal.



$$\therefore \quad \frac{K_1A(T_1-T)}{\ell_1} = \frac{K_2A(T-T_2)}{\ell_2},$$

where, A is the area of cross-section.

or, $K_1A(T_1-T)\ell_2 = K_2A(T-T_2)\ell_1$

or, $K_1T_1\ell_2 - K_1T\ell_2 = K_2T\ell_1 - K_2T_2\ell_1$

or, $(K_2\ell_1 + K_1\ell_2)T = K_1T_1\ell_2 + K_2T_2\ell_1$

$$\therefore \quad T = \frac{K_1T_1\ell_2 + K_2T_2\ell_1}{K_2\ell_1 + K_1\ell_2} = \frac{K_1\ell_2T_1 + K_2\ell_1T_2}{K_1\ell_2 + K_2\ell_1}.$$

5. **(4)** Here $TV^{\gamma-1}$ = constant

As $\gamma = \frac{5}{3}$, hence $TV^{2/3}$ = constant

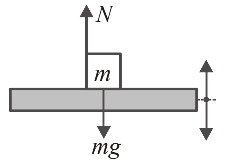
Now $T_1L_1^{2/3} = T_2L_2^{2/3}$ ($\because$ $V \propto L$);

Hence, $\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$

6. **(1)** The block can be detached from the platform, when it moves down. So

$mg - N = ma$

or $mg - 0 = m\omega^2 A$



$$\therefore \quad \omega = \sqrt{\frac{g}{A}}$$

and $T = 2\pi\sqrt{\frac{A}{g}} = 2\pi\sqrt{\frac{3.92\times10^{-3}}{g}}$

$= 0.1256$ s.

7. **(2)** Breaking stress $= \frac{\text{Force}}{\text{area}}$

The breaking force will be its own weight.

$F = mg = V\rho g$ = area × $\ell\rho g$

Breaking stress $= 6\times10^6 = \frac{\text{area}\times\ell\times\rho g}{\text{area}}$

or $\ell = \frac{6\times10^6}{3\times10^3\times10} = 200$ m.

8. **(4)** We know that efficiency of heat engine $= \text{I} - \frac{T_L}{T_H}$

Also, Efficiency of Heat engine $= \frac{\text{Work output}}{\text{Heat input}}$

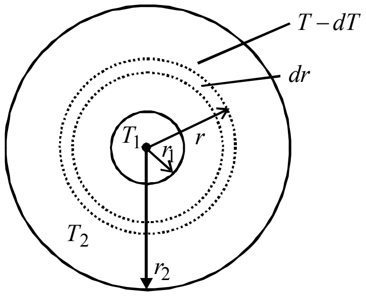
$$\therefore \quad 1 - \frac{T_L}{T_H} = \frac{W}{Q_S}$$

$$\Rightarrow \quad W = Q_S\left(1 - \frac{T_L}{T_H}\right)$$

$$= 6\times10^4\left(1 - \frac{127+273}{227+273}\right)$$

$= 1.2\times10^4$ cal

9. (4)



Consider a shell of thickness (dr) and of radius (r) and let the temperature of inner and outer surfaces of this shell be T and $(T - dT)$ respectively.

$\frac{dQ}{dt}$ = rate of flow of heat through it

$$= \frac{KA[(T-dT)-T]}{dr} = \frac{-KAdT}{dr}$$

$$= -4\pi K r^2 \frac{dT}{dr} \qquad (\because A = 4\pi r^2)$$

To measure the radial rate of heat flow, integration technique is used, since the area of the surface through which heat will flow is not constant.

Then, $$\left(\frac{dQ}{dt}\right)\int_{r_1}^{r_2}\frac{1}{r^2}dr = -4\pi K\int_{T_1}^{T_2}dT$$

$$\frac{dQ}{dt}\left[\frac{1}{r_1}-\frac{1}{r_2}\right] = -4\pi K\left[T_2 - T_1\right]$$

or $$\frac{dQ}{dt} = \frac{-4\pi K r_1 r_2 (T_2 - T_1)}{(r_2 - r_1)}$$

$$\therefore \quad \frac{dQ}{dt} \propto \frac{r_1 r_2}{(r_2 - r_1)}$$

10. (3) From Bernoulli's theorem,

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho g h = P_0 + \frac{1}{2}\rho v_2^2 + 0$$

$$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{0.16 + 2\times 10\times 0.2}$$

$= 2.03$ m/s

From equation of continuity

$$A_2 v_2 = A_1 v_1$$

$$\pi\frac{D_2^2}{4}\times v_2 = \pi\frac{D_1^2}{4}v_1$$

$$\Rightarrow \quad D_2 = \sqrt{\frac{v_1}{v_2}}D_1 = 3.55\times 10^{-3}\text{ m}$$

11. (3) $\ell_1 = \ell_2 = 45$ cm

The pressure must be same on both sides. Hence

$$\frac{\ell_1}{T_1} = \frac{\ell_2}{T_2} \Rightarrow \frac{\ell_1}{273} = \frac{\ell_2}{273+273} \Rightarrow \ell_1 = \frac{\ell_2}{2}$$

Also $\ell_1 + \ell_2 = 90$; $\therefore$ $\ell_1 = 30$cm and $\ell_2 = 60$cm

Now $$\frac{P_1(\ell_1 A)}{T_1} = \frac{P(\ell A)}{T}$$

or $$\frac{P_1\times 30}{273} = \frac{76\times 45}{273\times 81} \Rightarrow P_1 = 102.4\text{ cm}.$$

12. (2) Fundamental frequency,

$$f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{A\rho}}\left[\because v = \sqrt{\frac{T}{\mu}} \text{ and } \mu = \frac{m}{\ell}\right]$$

Also, $$Y = \frac{T\ell}{A\Delta\ell} \Rightarrow \frac{T}{A} = \frac{Y\Delta\ell}{\ell} \Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{\gamma\Delta\ell}{\ell\rho}} \quad ....(i)$$

$\ell = 1.5$ m, $\frac{\Delta\ell}{\ell} = 0.01$, $\rho = 7.7\times 10^3$ kg/m^3 (given)

$\gamma = 2.2\times 10^{11}$ N/m^2 (given)

Putting the value of ℓ, $\frac{\Delta\ell}{\ell}$, ρ and γ in eq^n. (i) we get,

$$f = \sqrt{\frac{2}{7}}\times\frac{10^3}{3}$$

or, $f \approx 178.2$ Hz

13. (3) Weight of the bowl = mg

$$= V\rho g = \frac{2}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g$$

Where D = Outer diameter

d = Inner diameter , ρ = Density of bowl

Weight of the liquid diplaced by the bowl

$$= V\sigma g = \frac{2}{3}\pi\left(\frac{D}{2}\right)^3\sigma g$$

where σ is the density of the liquid

For the floation

$$\frac{2}{3}\pi\left(\frac{D}{2}\right)^3\sigma g = \frac{2}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g$$

$$\Rightarrow \left(\frac{1}{2}\right)^3\times 1.2\times 10^3 = \left[\left(\frac{1}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]2\times 10^4$$

By solving we get d = 0.98 m.

14. (3) Young's modulus of rubber, Y_{rubber}

$$= \frac{F}{A}\times\frac{\ell}{\Delta\ell} \Rightarrow F = YA.\frac{\Delta\ell}{\ell}$$

On putting the values from question,

$$F = \frac{5\times 10^8\times 25\times 10^{-6}\times 5\times 10^{-2}}{10\times 10^{-2}}$$

$= 25\times 25\times 10 = 6250$N

kinetic energy = potential energy of rubber

$$\frac{1}{2}mv^2 = \frac{1}{2}F\Delta\ell$$

$$v = \sqrt{\frac{F\Delta\ell}{m}} = \sqrt{\frac{6250\times 5\times 10^{-2}}{5\times 10^{-3}}} = \sqrt{62500}$$

$= 25\times 10 = 250$ m/s

15. **(3)** Coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2} \qquad \text{(Where } Q_2 \text{ is heat removed)}$$

Given: $T_2 = 4°C = 4 + 273 = 277\ K$

$T_1 = 30°C = 30 + 273 = 303\ K$

$$\therefore \quad \beta = \frac{600 \times 4.2}{W} = \frac{277}{303 - 277}$$

$\Rightarrow$ W = 236.5 joule

Power $P = \frac{W}{t} = \frac{236.5\,\text{joule}}{1\,\text{sec}} = 236.5$ watt.

16. **(2)** $\frac{dQ}{dt} = KA\frac{\Delta T}{L}$

For the first rod, $\left(\frac{dQ}{dt}\right)_1 = \frac{3KA}{L}(100 - \theta)$

Similarly, $\left(\frac{dQ}{dt}\right)_2 = 2K\frac{A}{L}(\theta - 50)$

$\left(\frac{dQ}{dt}\right)_3 = K\frac{A}{L}(\theta - 20)$

Now, $\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2 + \left(\frac{dQ}{dt}\right)_3$

$\Rightarrow\ 3(100 - \theta) = 2(\theta - 50) + (\theta - 20) \Rightarrow \theta = 70°C$

17. **(4)** We have $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = a\omega\cos\left(\omega t + \frac{\pi}{6}\right)$

Maximum velocity = $a\omega$

According to question,

$\frac{a\omega}{2} = a\omega\cos\left(\omega t + \frac{\pi}{6}\right)$

or, $\cos\left(\omega t + \frac{\pi}{6}\right) = \frac{1}{2} = \cos 60°$ or $\cos\frac{\pi}{3}$

$\Rightarrow \omega t + \frac{\pi}{6} = \frac{\pi}{3}$

$\omega t = \frac{\pi}{3} - \frac{\pi}{6}$ or, $\omega t = \frac{\pi}{6}$

or, $\frac{2\pi}{T}.t = \frac{\pi}{6} \Rightarrow t = \frac{T}{12}$

18. **(1)** Here $\Delta \times 2\,Q = 0$ and $\Delta \times 2\,W = 0$. Therefore from first law of thermodynamics $\Delta U = \Delta \times 2\,Q + \Delta \times 2\,W = 0$

$\therefore$ Internal energy of the system with partition = Internal energy of the system without partition.

$$n_1 C_v T_1 + n_2 C_v T_2 = (n_1 + n_2) C_v T$$

$$\therefore T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

But $n_1 = \frac{P_1 V_1}{RT_1}$ and $n_2 = \frac{P_2 V_2}{RT_2}$

$$\therefore T = \frac{\frac{P_1V_1}{RT_1} \times T_1 + \frac{P_2V_2}{RT_2} \times T_2}{\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}}$$

$$= \frac{T_1T_2\,(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$$

19. **(1)** $V\rho g = 6\pi\eta r v + V\rho_\ell g$

$\Rightarrow\ Vg(\rho - \rho_\ell) = 6\pi\eta r v$

Also $Vg(\rho - \rho_\ell') = 6\pi\eta' r v'$

$\therefore\ v'\eta' = \frac{(\rho - \rho_\ell')}{(\rho - \rho_\ell)} \times v\eta$

$\Rightarrow\ v' = \frac{(\rho - \rho_\ell')}{(\rho - \rho_\ell)} \times \frac{v\eta}{\eta'}$

$= \frac{(7.8 - 1.2)}{(7.8 - 1)} \times \frac{10 \times 8.5 \times 10^{-4}}{13.2}$

$\therefore\ v' = 6.25 \times 10^{-4}$ cm/s

20. **(4)** $\frac{\Delta\ell_{steel}}{\Delta\ell_{brass}} = \frac{F_1\ell_1 / \pi r_1^2 Y_1}{F_2\ell_2 / \pi r_2^2 Y_2}$

$= \frac{2}{4} \times a \times \left(\frac{1}{b}\right)^2 \times \frac{1}{c}$

$= \frac{a}{2b^2c}$.

21. **(1)** $W_{AB} = 0, \qquad W_{BC} = P\Delta V = nR\Delta T = -nRT_0$

$W_{CA} = nRT\ell n\frac{V_f}{V_i} = nR(2T_0)\ell n2$

$Q_{BC} = nC_p\Delta T = \left(\frac{nR\gamma}{\gamma - 1}\right)T_0$

Efficiency, $\eta = \frac{W}{Q} = \left[\frac{2\ell n2 - 1}{\gamma/(\gamma - 1)}\right]$

22. **(3)** Given $\Delta\ell/\ell = 0.10\% = 0.001$ and $\Delta T = 100°C$

Now $\frac{\Delta\ell}{\ell} = \alpha\Delta T$

or $0.001 = \alpha \times 100$

or $\alpha = 10^{-5}/°C$

Further $\gamma = 3\alpha = 3 \times 10^{-5}/°C$

$\therefore\ \frac{\Delta V}{V} \times 100 = (3 \times 10^{-3})\ (100) = 0.30\%$

23. (2) For observer, tone of B will not change due to zero relative motion.
Observed frequency of sound produced by A

$$= 660\frac{(330-30)}{330} = 600\,Hz$$

$\therefore$ No. of beats = 600 – 596 = 4

24. (1) The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma RT}{M}}$

$$\therefore \frac{v_{O_2}}{v_{He}} = \sqrt{\frac{\gamma_{O_2}}{M_{O_2}} \times \frac{M_{He}}{\gamma_{He}}}$$

$$= \sqrt{\frac{1.4}{32} \times \frac{4}{1.67}} = 0.3237$$

$$\therefore v_{He} = \frac{v_{O_2}}{0.3237} = \frac{460}{0.3237} = 1421 \text{ m/s}$$

25. (3) The lengths of each rod increases by the same amount

$$\therefore \Delta\ell_a = \Delta\ell_s \Rightarrow \ell_1\alpha_a t = \ell_2\alpha_s t$$

$$\Rightarrow \frac{\ell_2}{\ell_1} = \frac{\alpha_a}{\alpha_s} \Rightarrow \frac{\ell_2}{\ell_1} + 1 = \frac{\alpha_a}{\alpha_s} + 1$$

$$\Rightarrow \frac{\ell_2 + \ell_1}{\ell_1} = \frac{\alpha_a + \alpha_s}{\alpha_s} \Rightarrow \frac{\ell_1}{\ell_1 + \ell_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}$$

26. (3) Work done = Surface tension × increase in area of the film

$W = S \times \Delta A$

Increase in area = Final area – initial area

$= 10 \times (0.5 + 0.1) - 10 \times 0.5 = 1 \text{ cm}^2$

$\therefore$ W = 72 × 2 × 1 = 144 erg

[$\because$ There are 2 free surfaces; $\therefore$ $\Delta A = 2 \times 1$].

27. (4) $\because$ Both wires are same materials so both will have same Young's modulus, and let it be Y.

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F}{A.(\Delta L/L)}, \text{ F = applied force}$$

A = area of cross-section of wire

$$\text{Now, } Y_1 = Y_2 \Rightarrow \frac{FL}{(A_1)(\Delta L_1)} = \frac{FL}{(A_2)(\Delta L_2)}$$

Since load and length are same for both

$$\Rightarrow r_1^2\Delta L_1 = r_2^2\Delta L_2, \left(\frac{\Delta L_1}{\Delta L_2}\right) = \left(\frac{r_2}{r_1}\right)^2 = 4$$

$\Delta L_1 : \Delta L_2 = 4:1$

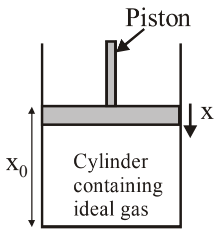
28. (3) $\frac{Mg}{A} = P_0$

$P_0 V_0^{\gamma} = PV^{\gamma}$

$Mg = P_0 A$ ...(1)

$P_0 A x_0^{\gamma} = PA(x_0 - x)^{\gamma}$

$$P = \frac{P_0 x_0^{\gamma}}{(x_0 - x)^{\gamma}}$$



Let piston is displaced by distance x

$$Mg - \left(\frac{P_0 x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right) A = F_{\text{restoring}}$$

$$P_0 A\left(1 - \frac{x_0^{\gamma}}{(x_0 - x)^{\gamma}}\right) = F_{\text{restoring}}$$

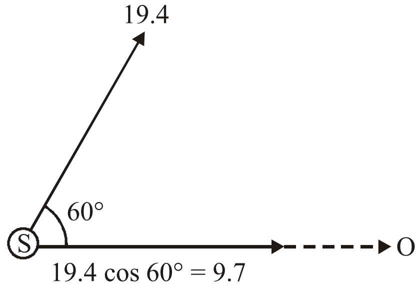
$$F = -\frac{\gamma P_0 A x}{x_0} \qquad [\because x_0 >> x]$$

$\therefore$ Frequency with which piston executes SHM.

$$f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A}{x_0 M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A^2}{M V_0}}$$

29. (1) Here, original frequency of sound, $f_0 = 100$ Hz

Speed of source $V_s = 19.4 \cos 60° = 9.7$



From Doppler's formula

$$f = f_0\left(\frac{V - V_0}{V - V_s}\right)$$

$$f = 100\left(\frac{V - 0}{V - (9.7)}\right)$$

$$f = 100\frac{V}{V\left(1 - \frac{9.7}{V}\right)}$$

$$f = 100\left(1 + \frac{9.7}{330}\right) = 103 \text{ Hz}$$

30. (4) Under adiabatic change

$$\frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}$$

or $T_2 = T_1 (P_1/P_2)^{\frac{1-\gamma}{\gamma}}$

$\therefore$ $T_2 = 300(4/1)^{\frac{1-(7/5)}{(7/5)}}$; $\gamma = 1.4 = 7/5$ for air

or $T_2 = 300(4)^{-2/7}$

PART B – CHEMISTRY

31. (1) $\overset{0}{4P} + 3KOH + 3H_2O \rightarrow 3KH_2\overset{+1}{PO_2} + \overset{-3}{PH_3}$

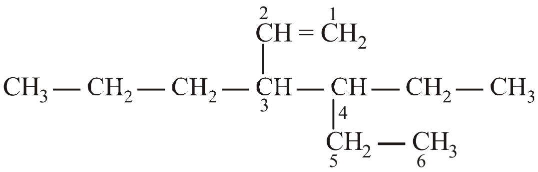
Hence P is both oxidised and reduced.

32. (1) Basic strength of the oxides increases in the order $Li_2O < Na_2O < K_2O < Rb_2O < Cs_2O$. The increase in basic strength is due to the decrease in I.E. and increase in electropositive character.

The melting points of the halides decrease in the order NaF > NaCl > NaBr > NaI, as the size of the halide ion increases. The decrease in melting point is due to increase in the covalent character with increase in the size of anion according to Fajan's rule.

33. (1) Since $I^{\ominus}$ is a better leaving group, so the reaction (1) is the best method.

34. (1) The given compound is



4-ethyl- 3-propyl hex – 1 – ene

35. (4) Sulphate of alkaline earth metal are sparingly soluble or almost not soluble in water whereas $BeSO_4$ is soluble in water due to high degree of solvation. $Be(OH)_2$ is insoluble in water but soluble in NaOH.

$BeO + 2NaOH \longrightarrow Na_2BeO_2 + H_2O$

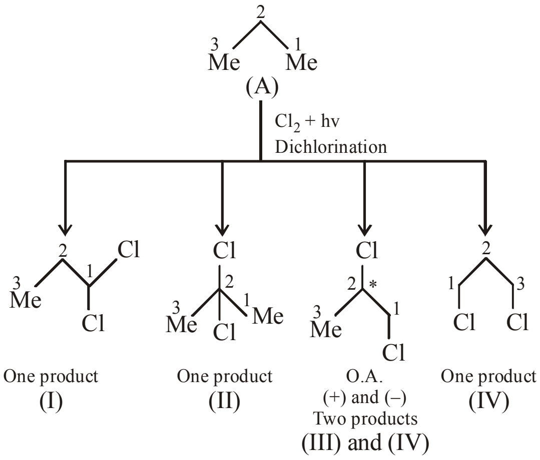
36. (1) Acidity : $H_3O^{\oplus} > Et\overset{\oplus}{O}H_2 > MeCOOH > H_2O > EtOH.$

Basicity and nucleophilicity :

$H_2O < MeOH < MeCOO^{\ominus} < \overset{\ominus}{O}H < EtO^{\ominus}$

i.e., (V) > (IV) > (III) > (II) > (I)

37. (2)



Total products on dichlorination of (1), i.e., numerical value of N is 5. On fractional distillation, the racemic mixture of III and IV cannot be separated but other structural isomers can be separated. So, the numerical value of M is 4.

Hence, the anwer is 5, 4.

38. (1) It is derivative of cyclohexane. The C-atom bearing – COOH group is to be assigned the number 1.

39. (3) The ozone layer, existing between 20 to 35 km above the earth's surface, shield the earth from the harmful U.V. radiations from the sun.

Depletion of ozone is caused by oxides of nitrogen

$N_2O + h\upsilon \longrightarrow \underset{\text{reactive nitric oxide}}{NO + N}$

$NO + O_3 \longrightarrow NO_2 + O_2$

$O_3 + h\upsilon \longrightarrow O_2 + O$

$NO_2 + O \longrightarrow NO + O_2$

$2O_3 + h\upsilon \longrightarrow 3O_2$ (Net reaction)

The presence of oxides of nitrogen increase the decomposition of O_3.

40. (4) Oxidizing agent itself, undergoes reduction during a redox reaction

$\overset{+3}{HAsO_2} + \overset{+2}{Sn} \longrightarrow \overset{0}{As} + \overset{+4}{Sn} + H_2O$

Hence, here $HAsO_2$ is acting as an oxidizing agent.

41. (3) $Ca(OH)_2$ is used for the softening of temporary hard water.

$Ca(OH)_2(aq) \xrightarrow{CO_2} \underset{\text{cloudiness}}{CaCO_3(s)} + H_2O(l)$

42. (3) The correct formula of inorganic benzene is $B_3N_3H_6$ so (4) is incorrect statement.

Boric acid (H_3BO_3 or B(OH)(OH)–OH) is a lewis acid so (1) is incorrect statement.

The coordination number exhibited by beryllium is 4 and not 6 so statement (2) is incorrect.

Both $BeCl_2$ and $AlCl_3$ exhibit bridged structures in solid state so (3) is correct statement.

43. (4) $CH_3 - CH_2 - \underset{}{\overset{CH_3}{\overset{|}{C}H}} - COOH$

44. (1)

45. (1) H_3BO_3 is a weak monobasic acid.

46. (2) Only 1-Alkynes form alkynides

47. (4)

$3\overset{0}{Br_2} + 6CO_3^{2-} + 3H_2O \longrightarrow 5\overset{-1}{Br} + \overset{+5}{Br}O_3^- + 6HCO_3^-$

Bromine is getting oxidised as well as reduced in this reaction.

48. (2)

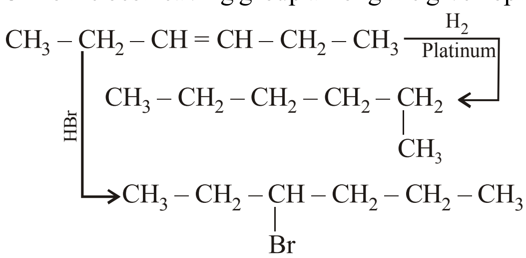
49. (1) 3° carbocations are most stable.

50. (4) In $CH \equiv CH$ triple bond consists of one σ and two π bonds.

51. (1)

52. (3) Cl^- is the best leaving group among the given option.

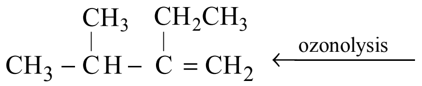
53. (1)



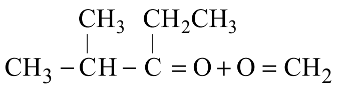
54. (4) Lower the E_{act}, of first step of chain propagation reaction, more easily the bond can be broken. Also this bond breaking will take place comparatively at a lower temperature.

55. (4) HI does not exhibit peroxide effect. HI bond although dissociates easily into iodine radicals, they being bigger in size are not much reactive but recombine together to form iodine molecule.

56. (1) Since the ozonolysis product, 2-methyl-3-pentanone, contains only six carbon atoms, while the alkene has seven carbon atoms, the other ozonolysis product should be CH_2O, the only carbonyl compound having one carbon atom. Hence, the structure of the alkene C_7H_{14} can be established as below



2−Ethyl−3−methyl−1−butene



2−Methyl−3pentanone

57. (4)

(1) (II), $I^{\ominus}$ is a better leaving group than $Br^{\ominus}$,

(2) (IV), 1° RX undergoes S_N2 reaction faster than 3° RX.

(3) (VI), Vinyl halide (V) does not undergo S_N1 or S_N2 reaction; (VI) is 1° RX, therefore undergoes S_N2.

58. (2) – I effect destablises carbocation and since inductive effect decreases with increasing length of carbon chain. Therefore (2) is the correct option.

59. (3) Statement-1 is true. However, lower oxidation state becomes more & more stable for heavier elements in *p*-block elements due to inert pair effect. Thus reason is false.

60. (1) The maximum valency of beryllium is +2 while that of aluminium is +3.

PART C – MATHEMATICS

61. (1) It is given that n coins have a head on both sides whereas (n + 1) coins are fair.

$$P\text{ (head)} = \frac{^{n}C_1}{^{2n+1}C_1}.1 + \frac{^{n+1}C_1}{^{2n+1}C_1}.\frac{1}{2} = \frac{31}{42}$$

$$\Rightarrow \quad \frac{n}{2n+1} + \frac{n+1}{2(2n+1)} = \frac{31}{42}$$

$\Rightarrow \quad 2n + n + 1 = (2n+1)(31/21)$

$\Rightarrow \quad 63n + 21 = 62n + 31$

$\Rightarrow \quad n = 10.$

62. (3) We know that, mean $= \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

$$\text{i.e., } 2.6 = \frac{1\times4 + 2\times5 + 3\times y + 4\times1 + 5\times2}{4+5+y+1+2}$$

or $31.2 + 2.6y = 28 + 3y$ or $0.4y = 3.2 \Rightarrow y = 8$

63. (1) Since $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines passing through origin (0, 0)

As, $lx + my = 1$

$$\Rightarrow \quad y = \frac{1 - lx}{m}$$

$$\text{So,} \quad ax^2 + 2hx\left(\frac{1-lx}{m}\right) + b\left(\frac{1-lx}{m}\right)^2 = 0$$

$\Rightarrow \quad (am^2 - 2hlm + bl^2)x^2 + 2(hm - bl)x + b = 0$

Suppose, (x_1, y_1) and (x_2, y_2) are the remaining two vertices of the triangle other than (0, 0).

$$\text{Then,} \quad x_1 + x_2 = \frac{2(bl - hm)}{am^2 - 2hlm + bl^2} \quad \text{(Sum of roots)}$$

$$\text{Similarly, } y_1 + y_2 = \frac{2(am - lh)}{am^2 - 2hlm + bl^2}$$

Therefore, centroid of the triangle

$$= \left(\frac{x_1 + x_2 + 0}{3}, \frac{y_1 + y_2 + 0}{3}\right)$$

$$= \left(\frac{x_1 + x_2}{3}, \frac{y_1 + y_2}{3}\right)$$

$$= \left(\frac{2}{3}\frac{(bl - hm)}{(am^2 - 2hlm + bl^2)}, \frac{2}{3}\frac{(am - lh)}{(am^2 - 2hlm + bl^2)}\right)$$

$$= \left(\frac{2}{3}\alpha, \frac{2}{3}\beta\right) \quad \text{(given)}$$

Hence,

$$\alpha = \frac{bl - hm}{am^2 - 2hlm + bl^2}, \ \beta = \frac{am - lh}{am^2 - 2hlm + bl^2}$$

64. (1) The equation of the chord joining (a sec θ, b tan θ) and (a sec ϕ, b tan ϕ) is

$$y - b\tan\phi = \frac{b\tan\theta - b\tan\phi}{a\sec\theta - a\sec\phi}(x - a\sec\phi)$$

which reduces to

$$\frac{x}{a}\cos\left(\frac{\theta-\phi}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta+\phi}{2}\right)$$

This passes through (ae, 0)

$$\therefore \quad e\cos\left(\frac{\theta-\phi}{2}\right) = \cos\left(\frac{\theta+\phi}{2}\right) \Rightarrow e = \frac{\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}$$

$$\Rightarrow \frac{e-1}{e+1} = \frac{\cos\left(\frac{\theta+\phi}{2}\right) - \cos\left(\frac{\theta-\phi}{2}\right)}{\cos\left(\frac{\theta+\phi}{2}\right) + \cos\left(\frac{\theta-\phi}{2}\right)}$$

[Applying componendo and dividendo theorem]

$\Rightarrow \dfrac{e-1}{e+1} = \tan\dfrac{\theta}{2}\tan\dfrac{\phi}{2}.$

65. (2) Here the given condition

$(a^2+b^2+c^2)\ p^2 - 2p\,(ab+bc+ca) + b^2+c^2+d^2 \le 0$

$\Rightarrow (ap-b)^2 + (bp-c)^2 + (cp-d)^2 \le 0$

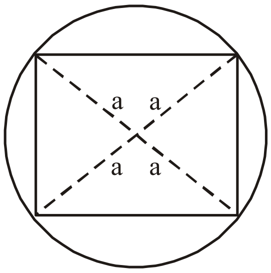
Since the squares can not be negative

$\therefore\ ap-b=0,\ bp-c=0,\ cp-d=0$

$\Rightarrow \dfrac{1}{p} = \dfrac{a}{b} = \dfrac{b}{c} = \dfrac{c}{d}$

$\therefore\ a,b,c,d$ are in G.P.

66. (3) Let a be the radius of the circle



$\therefore$ The area of the inscribed square

$= (a\sqrt{2})^2 = 2a^2$

$\therefore\ p_1 = \dfrac{2a^2}{\pi a^2} = \dfrac{2}{\pi}$

and $p_2 = \dfrac{\pi a^2 - 2a^2}{\pi a^2} = \dfrac{\pi-2}{\pi}$

$\therefore\ p_1 - p_2 = \dfrac{2}{\pi} - \left(\dfrac{\pi-2}{\pi}\right) = \dfrac{4}{\pi} - 1 > 0$

$\Rightarrow\ p_1 > p_2$

Also, $(p_1-p_2)(p_1+p_2) = \left(\dfrac{4-\pi}{\pi}\right)(1)$

but $3 < \pi < 4$

$\Rightarrow\ (p_1^2 - p_2^2) = \left(\dfrac{4-\pi}{\pi}\right) < \dfrac{1}{3}$

Hence, $(p_1^2 - p_2^2) < \dfrac{1}{3}$

67. (2) The equaition of AC is $\dfrac{x-2}{\cos 60°} = \dfrac{y-0}{\sin 60°} = r$

We have $AB = \sqrt{(3-2)^2 + (1-0)^2} = \sqrt{2}$. As AC is the new position of AB, therefore $AC = AB = \sqrt{2}$.

Thus, the coordinates of C are given by

$\dfrac{x-2}{\cos 60°} = \dfrac{y-0}{\sin 60°} = \sqrt{2} \Rightarrow x = 2 + \dfrac{1}{\sqrt{2}}$ and $y = \sqrt{\dfrac{3}{2}}$

Hence, the coordinates of C are $\left(2+\dfrac{1}{\sqrt{2}}, \sqrt{\dfrac{3}{2}}\right)$

68. (2) $(a_1a_3 - a_2a_2)^2 + (a_2a_4 - a_3a_3)^2 + \ldots\ldots + (a_{n-2}a_n - a_{n-1}a_{n-1})^2 \le 0$

(by lagrange's identity)

It is possible only if

$a_1a_3 - a_2a_2 = a_2a_4 - a_3a_3 = \ldots. = a_{n-2}\ a_n - a_{n-1}\ a_{n-1} = 0$

$\Rightarrow \dfrac{a_1}{a_2} = \dfrac{a_2}{a_3} = \dfrac{a_3}{a_4} = \ldots. = \dfrac{a_n}{a_{n-1}}$

Hence, $a_1, a_2, a_3, \ldots., a_n$ are in GP.

69. (1) $(1-x)(1-2x)(1-2^2x)(1-2^3x)\ldots(1-2^{15}x)$

$= (-(x-1))\left(-2\left(x-\dfrac{1}{2}\right)\right)\left(-2^2\left(x-\dfrac{1}{2^2}\right)\right)$

$\times\left(-2^3\left(x-\dfrac{1}{2^3}\right)\right)\ldots\left(-2^{15}\left(x-\dfrac{1}{2^{15}}\right)\right)$

$= 2^{1+2+3+\ldots+15}(x-1)\left(x-\dfrac{1}{2}\right)\left(x-\dfrac{1}{2^2}\right)$

$\times\left(x-\dfrac{1}{2^3}\right)\ldots\ldots\left(x-\dfrac{1}{2^{15}}\right)$

$= 2^{120}\left\{2x^{16} - \left(1+\dfrac{1}{2}+\dfrac{1}{2^2}+\dfrac{1}{2^3}+\ldots+\dfrac{1}{2^{15}}\right)x^{15} + \ldots\right\}$

$\therefore$ Coefficient of x^{15} in RHS

$= -2^{120}\left\{1+\dfrac{1}{2}+\dfrac{1}{2^2}+\dfrac{1}{2^3}+\ldots+\dfrac{1}{2^{15}}\right\}$

$= -2^{120}\left\{\dfrac{1\left(1-\left(\dfrac{1}{2}\right)^{16}\right)}{1-\dfrac{1}{2}}\right\}$

$= -2^{121}\left(1-\dfrac{1}{2^{16}}\right) = 2^{105} - 2^{121}$

70. (4) $\because\ \dfrac{\sum_{r=0}^{n-1} x^{2r}}{\sum_{r=0}^{n-1} x^r} = $ integer

$\Rightarrow \dfrac{\dfrac{1.(1-x^{2n})}{(1-x^2)}}{\dfrac{1.(1-x^n)}{(1-x)}} = $ integer

$\Rightarrow \quad \dfrac{1+x^n}{1+x} = \text{integer}$

So, n must be 1, 3, 5, 7, 9, 11, ...

$\therefore$ vertices are A (1, 7), B(3, 9) and C(5, 11)

Here, $(AB)^2 = 8, (BC)^2 = 8, (CA)^2 = 32$

$\therefore$ Triangle cannot be an equilateral

71. (3) We have,

$$\overline{X} = \frac{0.\,{}^nC_0 + 1.\,{}^nC_1 + 2.\,{}^nC_2 + ... + n.\,{}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n}$$

$$= \frac{\sum_{r=0}^{n} r.\,{}^nC_r}{\sum_{r=0}^{n} {}^nC_r}$$

$$= \frac{1}{2^n}\sum_{r=1}^{n} r.\frac{n}{r}\,{}^{n-1}C_{r-1}$$

$$= \left[\because \sum_{r=0}^{n} {}^nC_r = 2^n;\ {}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}\right]$$

$$= \frac{n}{2^n}\sum_{r=1}^{n} {}^{n-1}C_{r-1} = \frac{n}{2^n}2^{n-1} = \frac{n}{2} \quad \left[\because \sum_{r=1}^{n} {}^{n-1}C_{r-1} = 2^{n-1}\right]$$

$$\frac{1}{N}\sum f_i x_i^2 = \frac{1}{2^n}\sum_{r=0}^{n} r^2\,{}^nC_r$$

$$= \frac{1}{2^n}\sum_{r=0}^{n}[r(r-1)+r]\,{}^nC_r$$

$$= \frac{1}{2^n}\left\{\sum_{r=0}^{n} r(r-1)\,{}^nC_r + \sum_{r=0}^{n} r\,{}^nC_r\right\}$$

$$= \frac{1}{2^n}\left\{\sum_{r=2}^{n} r(r-1)\frac{n}{r}.\frac{n-1}{r-1}\,{}^{n-2}C_{r-2} + \sum_{r=1}^{n} r\frac{n}{r}\,{}^{n-1}C_{r-1}\right\}$$

$$= \frac{1}{2^n}\left\{n(n-1)\sum_{r=2}^{n} {}^{n-2}C_{r-2} + n\sum_{r=1}^{n} {}^{n-1}C_{r-1}\right\}$$

$$= \frac{1}{2^n}\left(n(n-1)2^{n-2} + n.\,2^{n-1}\right) = \frac{n(n-1)}{4} + \frac{n}{2}$$

$$\therefore \quad \text{Var}(X) = \frac{1}{N}\sum f_i x_i^2 - \overline{X}^2$$

$$= \frac{n(n-1)}{4} + \frac{n}{2} - \frac{n^2}{4} = \frac{n}{4}.$$

72. (1) We know that in an A.P.

$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = ...(i)$

$$\therefore \frac{1}{a_1a_n} + \frac{1}{a_2a_{n-1}} + \frac{1}{a_3a_{n-2}} + ... + \frac{1}{a_na_1}$$

$$= \frac{1}{a_1+a_n}\left[\frac{a_1+a_n}{a_1a_n} + \frac{a_1+a_n}{a_2a_{n-2}} + \frac{a_1+a_n}{a_3a_{n-2}} + ...+ \frac{a_1+a_n}{a_na_1}\right]$$

$$= \frac{1}{a_1+a_n}\left[\frac{a_1+a_n}{a_1a_n} + \frac{a_2+a_{n-1}}{a_2a_{n-2}} + \frac{a_3+a_{n-2}}{a_3a_{n-2}} + ... + \frac{a_n+a_1}{a_na_1}\right] \quad \text{[Using (i)]}$$

$$= \frac{1}{a_1+a_n}\left[\left(\frac{1}{a_1}+\frac{1}{a_n}\right)+\left(\frac{1}{a_2}+\frac{1}{a_{n-1}}\right)+\left(\frac{1}{a_3}+\frac{1}{a_{n-2}}\right)+ ....+\left(\frac{1}{a_1}+\frac{1}{a_n}\right)\right]$$

$$= \frac{2}{a_1+a_n}\left[\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+......+\frac{1}{a_n}\right]$$

(Each term occurs twice).

73. (1) The parametric equation of a line through A is

$$\frac{x+5}{\cos\theta} = \frac{y+4}{\sin\theta} = r$$

Let $AB = r_1$, $AC = r_2$ and $AD = r_3$

Then the coordinates of B, C, D are

$(-5 + r_1\cos\theta, -4 + r_1\sin\theta)$, $(-5 + r_2\cos\theta, -4 + r_2\sin\theta)$ and $(-5 + r_3\cos\theta, -4 + r_3\sin\theta)$ respectively

Now B lies on the line $x + 3y + 2 = 0$

$\therefore \quad -5 + r_1\cos\theta + 3(-4 + r_1\sin\theta) + 2 = 0$

$$\frac{15}{r_1} = \cos\theta + 3\sin\theta$$

$\because$ C lies on $2x + y + 4 = 0$

$\therefore \quad 2(-5 + r_2\cos\theta) + (-4 + r_2\sin\theta) + 4 = 0$

$$\Rightarrow \frac{10}{r_2} = 2\cos\theta + \sin\theta$$

$\because$ D lies on $x - y - 5 = 0$

$\therefore \ -5 + r_3\cos\theta + 4 - r_3\sin\theta - 5 = 0 \Rightarrow \dfrac{6}{r_3} = \cos\theta - \sin\theta$.

From the given condition, $\left(\dfrac{15}{r_1}\right)^2 + \left(\dfrac{10}{r_2}\right)^2 = \left(\dfrac{6}{r_3}\right)^2$

We get, $(\cos\theta + 3\sin\theta)^2 + (2\cos\theta + \sin\theta)^2 = (\cos\theta - \sin\theta)^2$

$\Rightarrow (2\cos\theta + 3\sin\theta)^2 = 0 \quad \Rightarrow \tan\theta = -\dfrac{2}{3}$

$\therefore$ Equation of the line is

$$y + 4 = -\frac{2}{3}(x+5) \Rightarrow 2x + 3y + 22 = 0$$

74. (2) We have, $\dfrac{1}{x_1}, \dfrac{1}{x_2}, \dfrac{1}{x_3}, ..., \dfrac{1}{x_n}$ are in A.P.

$$\therefore \frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{x_3} - \frac{1}{x_2} = ... = \frac{1}{x_n} - \frac{1}{x_{n-1}} = d \quad \text{(say)}$$

$\therefore \frac{x_1 - x_2}{x_1 x_2} = \frac{x_2 - x_3}{x_2 x_3} = ... = \frac{x_{n-1} - x_n}{x_{n-1} x_n} = d$

Now, $x_1x_2 + x_2x_3 + ... + x_{n-1}x_n$

$= \frac{1}{d}[x_1 - x_2 + x_2 - x_3 + ... + x_{n-1} - x_n] = \frac{x_1 - x_n}{d}$

But $\frac{1}{x_n} = \frac{1}{x_1} + (n-1)d$

$\therefore \frac{x_1 - x_n}{x_1 x_n} = (n-1)d$ or $\frac{x_1 - x_n}{d} = (n-1)x_1x_n$

$\therefore x_1x_2 + x_2x_3 + ... + x_{n-1}x_n = (n-1)x_1x_n$

75. **(2)** Here, equations of pair of straight lines are

$x^2 - 2mxy - y^2 = 0$ ...(i)

$x^2 - 2nxy - y^2 = 0$ ...(ii)

Therefore, equations of bisectors of these lines are

$mx^2 + 2xy - my^2 = 0$ ...(iii)

$nx^2 + 2xy - ny^2 = 0$ ...(iv)

But according to the condition (i) and (iv), and (ii) and (iii) must be coincident,

$\therefore \frac{n}{1} = \frac{2}{-2m} = \frac{-n}{-1} \Rightarrow mn = -1$

76. **(4)** Mid-point of P(2, 3) and Q(4, 5) = (3, 4)

Slope of PQ = 1, Slope of the line L = – 1

Mid-point (3, 4) lies on the line L.

Equation of line L,

$y - 4 = -1(x - 3) \Rightarrow x + y - 7 = 0$ ...(i)

Let image of point R(0, 0) be $S(x_1, y_1)$

Mid-point of RS = $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$

Mid-point $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$ lies on the line (i)

$\therefore x_1 + y_1 = 14$ ...(ii)

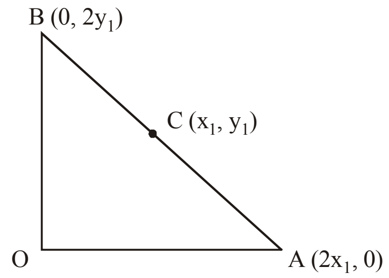
Slope of RS = $\frac{y_1}{x_1}$; Since RS $\perp$ line L

$\therefore \frac{y_1}{x_1} \times (-1) = -1 \quad \therefore x_1 = y_1$ ...(iii)

From (ii) and (iii), $x_1 = y_1 = 7$

Hence the image of R = (7, 7)

77. **(1)** If (x_1, y_1) is the circumcentre of the ΔABO, then (x_1, y_1) is the mid-point of the hypotenuse AB. Therefore A is $(2x_1, 0)$ and B is $(0, 2y_1)$.



Also $x_1 + y_1 - x_1y_1 + m\sqrt{x_1^2 + y_1^2} = 0$ ....(1)

Inradius $= \sqrt{2^2 + 2^2 - 4} = 2$

i.e. $\frac{\text{Area of } \Delta ABC}{\left(\frac{OA + OB + AB}{2}\right)} = 2$ $\left[\text{using } r = \frac{\Delta}{s}\right]$

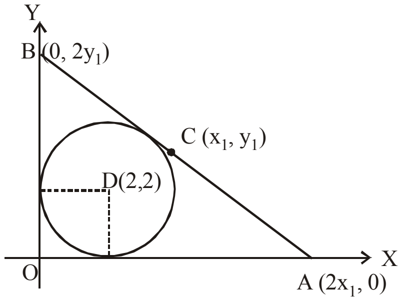
$\Rightarrow \frac{\frac{1}{2}(2x_1)(2y_1)}{x_1 + y_1 + \sqrt{x_1^2 + y_1^2}} = 2$

$\Rightarrow x_1 + y_1 - x_1y_1 + \sqrt{x_1^2 + y_1^2} = 0$ ....(2)

From eqs. (1) and (2), we get m = 1.

ALTERNATE:

The equation of the line AB is



$\frac{x}{2x_1} + \frac{y}{2y_1} = 1 \Rightarrow xy_1 + x_1y - 2x_1y_1 = 0$

[$C(x_1, y_1)$ is mid point of AB, i.e. circumcentre of ΔAOB]

Line AB touches the incircle

$\therefore$ Perpendicular from centre = radius

$\therefore \frac{2x_1 + 2y_1 - 2x_1y_1}{\sqrt{x_1^2 + y_1^2}} = \pm 2$

Since D(2, 2) lies below the line AB.

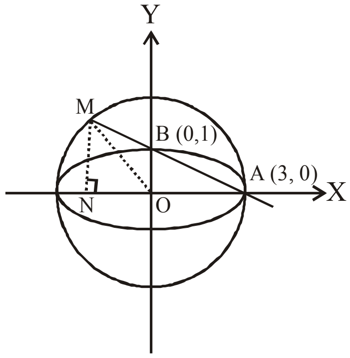
$\therefore 2x_1 + 2y_1 - 2x_1y_1 < 0$, hence negative sign must be taken. The required locus is $x + y - xy + \sqrt{x^2 + y^2} = 0$

78. **(4)** Equation of the ellipse is

$$\frac{x^2}{9}+\frac{y^2}{1}=1$$

An end of the major axis A be say (3, 0) and an end of the minor axis B be say (0, 1).

So, Equation of AB is $\frac{x}{3}+\frac{y}{1}=1$ ... (1)



Equation of the auxiliary circle is $x^2+y^2=9$ .... (2)
Solving the equations (1) and (2), we get

$$x^2+\left(1-\frac{x}{3}\right)^2=9 \Rightarrow x^2+1+\frac{x^2}{9}-\frac{2x}{3}=9$$

$$\Rightarrow \frac{10x^2}{9}-\frac{2x}{3}-8=0$$

$$\Rightarrow 5x^2-3x-36=0 \Rightarrow (5x+12)(x-3)=0$$

$$\therefore x=-\frac{12}{5} \Rightarrow y=1-\frac{1}{3}\left(-\frac{12}{5}\right)=\frac{9}{5}$$

$\therefore$ Coordinates of M $\equiv \left(-\frac{12}{5},\frac{9}{5}\right)$

Area of Δ AOM $=\frac{1}{2}$. OA . MN $=\frac{1}{2}\times 3\times\frac{9}{5}=\frac{27}{10}$

79. **(1)** (x_i, y_i), i = 1, 2, 3, 4 lies on $xy=c^2 \Rightarrow y_i=\frac{c^2}{x_i}$

Now the point (x_i, y_i) lies on

$$x^2+y^2=a^2 \Rightarrow x_i^2+\frac{c^4}{x_i^2}=a^2$$

$$\Rightarrow x_i^4-a^2x_i^2+c^4=0$$

Its roots are x_1, x_2, x_3, x_4

$$\therefore x_1+x_2+x_3+x_4=0$$

$$x_1x_2+x_1x_3+x_1x_4+x_2x_3+x_2x_4+x_3x_4=a^2$$

$$x_1x_2x_3+x_1x_2x_4+x_1x_3x_4+x_2x_3x_4=0$$

$$x_1x_2x_3x_4=c^4$$

Clearly (3) is not correct

Now $y_1y_2y_3y_4=\frac{c^2}{x_1}\cdot\frac{c^2}{x_2}\cdot\frac{c^2}{x_3}\cdot\frac{c^2}{x_4}=c^4$

Clearly (4) is not correct.

$$y_1+y_2+y_3+y_4=\frac{c^2(\Sigma x_1x_2x_3)}{x_1x_2x_3x_4}=0$$

Clearly (2) is not correct.

80. **(1)** We have limit $=\lim_{x\to 0}\left[\frac{a}{x}-\frac{\cos(x/a)}{\sin(x/a)}\right]$

$$=\lim_{x\to 0}\left[\frac{a\sin(x/a)-x\cos(x/a)}{x\sin(x/a)}\right]$$

$$=a\lim_{x\to 0}\left[\frac{a\sin(x/a)-x\cos(x/a)}{x^2}\right]\times\frac{(x/a)}{\sin(x/a)}$$

$$=a\lim_{x\to 0}\left[\frac{a\sin(x/a)-x\cos(x/a)}{x^2}\right]\left(\frac{0}{0}\text{ form}\right)$$

$$=a\lim_{x\to 0}\left[\frac{\cos(x/a)-\cos(x/a)+(x/a)\sin(x/a)}{2x}\right]$$

$=0$

81 **(3)** We consider following truth table.

p	q	$\sim p$	$\sim q$	$p\wedge q$	$p\vee q$	$(\sim(p\vee q))$	$(p\wedge q)\wedge(\sim(p\vee q))$
T	T	F	F	T	T	F	F
T	F	F	T	F	T	F	F
F	T	T	F	F	T	F	F
F	F	T	T	F	F	T	F

Clearly last column of the above truth table contains only F.
Hence $(p\wedge q)\wedge(\sim(p\vee q))$ is a contradiction

82. **(3)** $z_r=\cos\frac{r\alpha}{n^2}+i\sin\frac{r\alpha}{n^2}$

$$z_1=\cos\frac{\alpha}{n^2}+i\sin\frac{\alpha}{n^2};$$

$$z_2=\cos\frac{2\alpha}{n^2}+i\sin\frac{2\alpha}{n^2};$$

$$z_n=\cos\frac{n\alpha}{n^2}+i\sin\frac{n\alpha}{n^2}$$

consider $\lim_{n\to\infty}(z_1z_2z_3......z_n)$

$$= \lim_{n\to\infty}\left[\cos\left\{\frac{\alpha}{n^2}(1+2+3+....+n)\right\} + i\sin\left\{\frac{\alpha}{n^2}(1+2+3+...+n)\right\}\right]$$

$$= \lim_{n\to\infty}\left[\cos\left\{\frac{\alpha\left(1+\frac{1}{n}\right)}{2}\right\} + i\sin\left\{\frac{\alpha\left(1+\frac{1}{n}\right)}{2}\right\}\right]$$

$$= \cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2} = e^{\frac{i\alpha}{2}}$$

83. (1) $\frac{8}{4} = 2, \frac{64}{4} = 16$; but 4 is not prime.

Hence $P \wedge Q \to R$ is false

$\frac{(6)^2}{12} = \frac{36}{12} = 3$; but 12 is not prime

Hence $Q \to R$ is false

$\frac{(4)^2}{8} = \frac{16}{8} = 2$; $\frac{4}{8}$ is not an integer

Hence $Q \to P$ is false

Now consider m is prime and m divides n^2, then two cases arise as below:

Case 1 : If m = n,

then obviously m will divide n.

Case 2 : If $m \neq n$,

As m divides n^2. So m is a factor of n, then clearly m will divide n

Hence $Q \wedge R \to P$ is true

84. (1) We divide the number in three groups

3k + 1 type {1, 4, 7, ..., 2005}

3k + 2 type {2, 5, 8, ..., 2006}

3k + 3 type {3, 6, 9, ..., 2007}

$x^3 + y^3$ is divisible by 3 if x and y both belong to 3rd group or one of them belongs to the first group and the other to the second group.

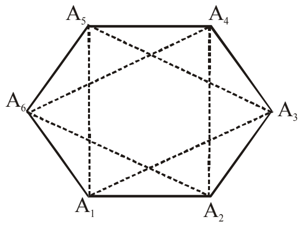
So, favourable number of cases $= {}^{669}C_2 + 669 \times 669$

Total number of cases $= {}^{2007}C_2$

$\therefore$ Desired probability $= \dfrac{\frac{669 \times 668}{2} + 669 \times 669}{\frac{2007 \times 2006}{2}}$

$$= \frac{669 \times 2006}{2007 \times 2006} = \frac{1}{3}$$

85. (3) Three vertices can be selected in ${}^6C_3$ ways.



The only equilateral triangles possible are $A_1A_3A_5$ and $A_2A_4A_6$

$$p = \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$$

86. (4) Since the ([P + 1], [P]) lies inside the circle

$x^2 + y^2 - 2x - 15 = 0$ (But $[x + n] = [x] + n, n \in N$)

$\therefore$ $[P + 1]^2 + [P]^2 - 2[P + 1] - 15 < 0$

$([P] + 1)^2 + [P]^2 - 2([P] + 1) - 15 < 0$

$2[P]^2 - 16 < 0$, $[P]^2 < 8$ ...(1)

From the second circle

$([P] + 1)^2 + [P]^2 - 2([P] + 1) - 7 > 0$

$\Rightarrow$ $2[P]^2 - 8 > 0$, $[P]^2 > 4$ ...(2)

From (1) and (2), $4 < [P]^2 < 8$, which is not possible

$\therefore$ For no values of 'P' the point will be within the region.

87. (4) Let the point of intersection of tangents at points α and β, where $\alpha + \beta = \phi$ (constant), be (h, k),

Then chord of contact is $\frac{hx}{a^2} + \frac{ky}{b^2} = 1$ …(1)

Also, the line joining points α and β is

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$$

or $\frac{x}{a}\cos\frac{\phi}{2} + \frac{y}{b}\sin\frac{\phi}{2} = \cos\frac{\alpha-\beta}{2}$ …(2)

(1) and (2) are same lines, so

$$\frac{h}{a\cos\frac{\phi}{2}} = \frac{k}{b\sin\frac{\phi}{2}} \Rightarrow k = \frac{b}{a}\tan\frac{\phi}{2}h$$

So, locus is $y = \frac{b}{a}\tan\frac{\phi}{2}x$ (Straight line)

88. (3) Let the G.P. be a, ar, ar^2, ...

$S = a + ar + ar^2 +$ ....upto $2n$ terms

$$= \frac{a(r^{2n} - 1)}{r - 1}$$

The series formed by taking term occupying odd places is $S_1 = a + ar^2 + ar^4 +$ .... to n terms

$\Rightarrow S_1 = \dfrac{a(r^{2n}-1)}{r^2-1}$

Now, $S = 5S_1$

or $\dfrac{a(r^{2n}-1)}{r-1} = 5\dfrac{a(r^{2n}-1)}{r^2-1}$

$\Rightarrow \quad 1 = \dfrac{5}{r+1}$

$\Rightarrow r+1 = 5 \therefore r = 4$

89. **(4)** Putting $x = \dfrac{1}{y}$, we get

$$L = \lim_{y\to 0}\left(\frac{a_1^y + a_2^y + ... + a_n^y}{n}\right)^{n/y} \quad (\because x\to\infty \Rightarrow y\to 0)$$

$$\therefore \log_e L = \lim_{y\to 0}\frac{n}{y}.\log_e\left\{\frac{1}{n}\left(a_1^y + a_2^y + ... + a_n^y\right)\right\} \left(\frac{0}{0}\right) \text{ form}$$

$$= n\lim_{y\to 0}\frac{\left(\dfrac{a_1^y \log a_1 + a_2^y \log a_2 + ... + a_n^y \log a_n}{a_1^y + a_2^y + ... + a_n^y}\right)}{1}$$

[using L′Hopital rule]

$$= n.\frac{\log(a_1 a_2 ....a_n)}{n}$$

$\therefore \log L = \log(a_1.a_2.....a_n) \Rightarrow L = a_1.a_2.a_3......a_n$

90. **(3)** $\sim \{(p \vee (\sim q)) \wedge q\}$

$= (\sim (p \vee (\sim q))) \vee (\sim q)$ (By De Morgan's Law,)

$= ((\sim p) \wedge (\sim (\sim q))) \vee (\sim q)$ [Using De Morgan's law]

$= (\sim p \wedge q) \vee (\sim q)$ $[\because \sim(\sim q) = q]$