

# Algebra

Topics	Contents	Prerequisites	Optional additions
Sequence and series	AP, GP, HP, special sequences, sum of infinite series, converting to known forms, binomial coefficients and identities, Fibonacci and bell numbers etc	AP-GP up to 10 <sup>th</sup> std	expressing with recurrences, expressions of some famous series, convergence, Riemann zeta
Systems of linear equations	Solving multiple variable linear equations (Elimination method)	Equations in 2/3 variables, criterion of consistency etc	Use of matrices, determinants, rank, solution criterion etc
Trigonometric Equations	Unit circle, trigonometric identities, Inequalities, behavior in intervals on real line	Trigo Identities and definitions up to std 10	Euler's formula, identities for some standard series.
Polynomial factorizations	Remainder theorem, Factor theorem, Division algorithm, Descartes' rule of signs, Vieta's formulas for polynomial coefficients, Sophie Germain identity, Newton's identities	Quadratic equations and their graphs, criterion for real roots.	(Expected comfort in splitting any polynomial into 2 factors if one root is given, by the end)  Use in recurrence relations
Roots and irreducibility	Extension of Descartes', Eisenstein criterion,	Changes in graph of function on operation of constants, basic derivatives	Complex numbers, roots of unity, Newton Polygon, $F_p$ ' cyclotomic polynomials
Inequalities	RMS-AM-GM-HM, Weighted means, Cauchy Schwarz, Engel form, Chebyshev's, Holder, Minkowski, Jensen, Schur's, Muirhead, Rearrangement, completing the square method, trigonometric substitutions	Real numbers and their properties, one should be comfortable factorizing expressions	Triangle inequality in complex numbers, Use of Lagrange Multipliers method, conversion to Geometric/Complex number problems
Functional Equations	Solving functional equations, mostly practice	Basic idea of functions	Cyclic functions and mobius inversion
Complex Numbers	Fundamental Theorem, De Moivre's theorem, Basic uses of roots of unity,	a+ib notation, interpreting complex numbers as vectors (hence basic vectors, dot product	Use in geometry (bash)

# Number Theory

Topics	Contents	Prerequisites	Optional additions
Principle of Mathematical Induction	Multiple variants uses (strong form etc), mostly examples		Construction of Natural numbers with Peano's axioms, relation with axiom of choice
Divisibility criterions	Divisibility by special numbers, examples and applications, GCD, LCM, rational numbers, recurring decimals	Basic arithmetic, Euclid's algorithm	numbers with multiple expansions in a base, continued fractions
Greatest Integer Function	Properties, problems	Real numbers, general idea	Egyptian fraction etc
Arithmetic Functions	No. of divisors, sum of divisors, Mobius function	Number factorization	Mobius formula and inversion in general poset.
Euler's phi function	coprime numbers, Bezout's identity, lemma, properties of phi and uses in Modular arithmetic	Definitions of coprime numbers, one should be comfortable seeing divisibility by observation.	Dirichlet's convolution, general Mobius inversion in partially ordered sets
Modular arithmetic	Congruences, Theorems of Fermat, Euler, Wilson, Chinese Remainder theorem	One should be comfortable finding remainders of divisions of numbers by observations	Primitive roots, Quadratic residues, Lifting the exponent lemma, primality tests in computation.
Diophantine equations	Infinite Descent , Pell's equation, applications	General idea of factorizing polynomials	Abc conjecture, collatz conjecture
Pythagorean triplets	Generating formula, primitive triples		

## Geometry

Topics	Contents	Prerequisites	Optional additions
Congruent-similar triangles	Problems based on congruence/ similarity, construction of congruent triangles etc	Tests for similarity, congruence up to std 10, general idea of writing proofs in geometry	Desargue's theorem
Concurrence and collinearity	General discussion, Stewart's theorem, Menelaus theorem, Ceva's theorem, Apollonius circle	9 <sup>th</sup> /10 <sup>th</sup> std geometry general idea	Introduction to Projective geometry, harmonic pencils
Triangle special points	Centroid, incenter, excenters, orthocenter, circumcenter, Gergonne and Nagel point, Lemoine point, Neuberg and Brocard points, Miquel points.  Orthic triangle, Pedal triangle.	General triangle geometry	Morley's theorem, Barycentric coordinates, Bash with complex numbers, Schiffler's point
Triangle special lines	Medians, Angle bisectors, altitudes, symmedians, Euler line, Simson line, Newton-Gauss line	--	Bash with coordinate/ barycentric geometry
Triangle Special circles	incircle, excircle, mixtilinear circles, Nine point circle and generalization	Construction of excircle, incircle.	Eleven point conic
Dilations and inversion	Homothety, Results about inversion, dilation, isogonal and isotomic conjugates, <b>Spiral similarity</b> , general geometric transformations (folding, stretching etc)	Similarity of triangles, one should be comfortable imagining rotations and dilations	
Circles and Cyclic Quadrilaterals	Pole-polars, Ptolemy's theorem, Casey theorem, Butterfly theorem, Power of point, radical axis, Orthogonal circles	Properties of angles in arc, tangent secant theorem, inscribed angle	Brahmagupta quadrilaterals, general orthogonal curves explanation with calculus
Constructions	All constructions involving advanced analysis of geometric properties, includes all above topics	Construction up to std 10	

# Combinatorics

Topics	Contents	Prerequisites	Optional additions
Elementary counting problems	Permutations and Combinations, Catalan numbers, Bell and Sterling numbers, solutions for closed forms with multiple methods.	Factorials, Binomial coefficients.	Gamma and beta functions
Pigeonhole principle	Applications of PHP in problems (mostly practice)	--	Applications in chromatic number, independent sets, general Matroids
Invariants	Spotting invariants and using them	--	Random walks in Markov chains
Coloring proofs	Chessboard and other colorings, Domino-Triomino covering problems.	--	
Graphs and graph colorings	graph colorings and chromatic number, complete graphs, spanning trees, forests, bipartite graphs, triangle free graphs (k-clique free graphs in general)	General identities about graphs, notion of connected, acyclic graph, edge set-vertex set-degree relations, euler's formula	meaning in independent sets and matroids, graph traversals (DFS-BFS), strongly connected components in directed graphs, Dijkstra- Kruskal algorithms, min cost spanning trees etc
Generating functions	Guessing/deriving generating function for a problem, correctness argument, special numbers as coefficients in generating functions, use of partial fractions to obtain closed forms	Geometric progressions and infinite sum, basic polynomial factorization	Bessel Functions, Cauchy product of series
Recurrence relations	Relation with generating functions, solving linear recurrence relations, obtaining recurrences from problems, mostly practice	Generating functions, if want proofs.	Bessel function and applications in recurrence closed form derivations
Probability	General probability discussion, mostly based on elementary counting like binomial counting or tournament graphs	Idea of probability up to std 10	Markov Chains, Irreducibility and aperiodicity, uses of transition matrix,
Greedy algorithms Divide & conquer	Fibonacci numbers, writing proof of correctness, Interval scheduling problem, job scheduling, deadline-profit		Introduction to dynamic programming