

**PART (C): MATHEMATICS**

**SECTION I: (SINGLE CHOICE QUESTIONS)**

This section contains **30 multiple choice questions**. Each question has four choices (1), (2), (3) and (4) out of which **ONLY ONE is correct**

61. The sum of three numbers in AP is  $-3$  and their product is  $8$ . Then, sum of squares of the numbers is  
 (1) 9 (2) 10 (3) 21 (4) 12

61. (3)

Let the three numbers in AP are  $a-d, a, a+d$ .

$$\therefore (a-d) + a + (a+d) = -3$$

$$\Rightarrow 3a = -3$$

$$\Rightarrow a = -1$$

and  $(a-d)(a)(a+d) = 8$

$$\Rightarrow (-1)\{(-1)^2 - d^2\} = 8$$

$$\Rightarrow d^2 = 9$$

$$\Rightarrow d = \pm 3$$

If  $d = 3$ , then numbers are  $-4, -1$  and  $2$ .

If  $d = -3$ , then numbers are  $2, -1$  and  $-4$ .

$$\begin{aligned} \text{Hence, the sum of squares of numbers} &= (-4)^2 + (-1)^2 + (2)^2 \\ &= 16 + 1 + 4 = 21 \end{aligned}$$

62. If we insert two numbers between 3 and 81 so that the resulting sequence is GP. Then, the numbers are

- (1) 9, 27 (2) 8, 27 (3) 9, 25 (4) None of these

62. (1)

Let the two numbers are  $a$  and  $b$ , then  $3, a, b, 81$  are in GP.

$$\therefore \text{nth term } T_n = AR^{n-1}$$

$$\therefore 81 = 3R^{4-1}$$

$$\Rightarrow R^3 = \frac{81}{3} = 27$$

$$\Rightarrow R^3 = 3^3$$

$$\Rightarrow R = 3$$

$$\therefore a = AR = 3 \times 3 = 9, b = AR^2 = 3 \times 3^2 = 27$$

63. If the ratio of the sum of  $n$  terms of two AP's be  $(7n+1):(4n+27)$ , then the ratio of their  $11^{\text{th}}$  terms will be

- (1) 2 : 3 (2) 3 : 4 (3) 4 : 3 (4) 5 : 6

63. (3)

Let  $S_n$  and  $S'$  be the sums of  $n$  terms of two AP's and  $T_{11}$  and  $T'_{11}$  be the respective  $11^{\text{th}}$  term, then

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']} = \frac{7n+1}{4n+27} \quad (\text{given})$$

$$\Rightarrow \frac{a + \frac{(n-1)}{2}d}{a' + \frac{(n-1)}{2}d'} = \frac{7n+1}{4n+27}$$

Now, put  $n = 21$ , we get

$$\frac{a+10d}{a'+10d'} = \frac{T_1}{T'_{11}} = \frac{148}{111} = \frac{4}{3}$$

**64.** If the AM and GM of roots of a quadratic equations are 8 and 5 respectively, then the quadratic equation will be

(1)  $x^2 - 16x - 25 = 0$

(2)  $x^2 - 8x + 5 = 0$

(3)  $x^2 - 16x + 25 = 0$

(4)  $x^2 + 16x - 25 = 0$

**64.** (3)

Given that, AM = 8, GM = 5, if  $\alpha, \beta$  are the roots of quadratic equation, then the required quadratic equation is

$$x^2 - x(\alpha + \beta) + \alpha\beta = 0 \quad \dots(i)$$

Here, AM =  $\frac{\alpha + \beta}{2} = 8 \Rightarrow \alpha + \beta = 16$

and GM =  $\sqrt{\alpha\beta} = 5 \Rightarrow \alpha\beta = 25$

From Eq. (i),

$$x^2 - 16x + 25 = 0$$

**65.** If  $a + 2b + 3c = 12$ , ( $a, b, c \in R^+$ ), then  $ab^2c^3$  is

(1)  $\geq 2^3$

(2)  $\geq 2^6$

(3)  $\leq 2^6$

(4) None of these

**65.** (3)

Given that,  $a + 2b + 3c = 12$

and  $a, b, c$  are positive real numbers.

Now, AM  $\geq$  GM

$$\Rightarrow \frac{a+b+b+c+c+c}{6} \geq \sqrt[6]{ab^2c^3}$$

$$\Rightarrow \frac{a+2b+3c}{6} \geq \sqrt[6]{ab^2c^3}$$

$$\Rightarrow ab^2c^3 \leq 2^6$$

**66.**  $1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \dots \dots \infty$  is equal to

(1) 3

(2) 6

(3) 9

(4) none of these

**66.** (2)

67. The solution of the equation  $\sin x + \sin 3x + \sin 5x = 0$  is  
 (1)  $\frac{n\pi}{3}, n\pi \pm \frac{\pi}{3}$       (2)  $\frac{n\pi}{2}, n\pi \pm \frac{\pi}{3}$       (3)  $\frac{n\pi}{3}, n\pi \pm \frac{\pi}{4}$       (4) None of these

67. (1)

$$\begin{aligned} \sin x + \sin 3x + \sin 5x &= 0 \\ \Rightarrow (\sin 5x + \sin x) + \sin 3x &= 0 \\ \Rightarrow 2\sin \frac{5x+x}{2} \cos \frac{5x-x}{2} + \sin 3x &= 0 && \left[ \because \sin C + \sin D = 2\sin \left( \frac{C+D}{2} \right) \cos \left( \frac{C-D}{2} \right) \right] \\ \Rightarrow 2\sin 3x \cos 2x + \sin 3x &= 0 \\ \Rightarrow \sin 3x(2\cos 2x + 1) &= 0 \end{aligned}$$

Either  $\sin 3x = 0$  or  $2\cos 2x + 1 = 0$   
 When  $\sin 3x = 0$ ,

Then,  $3x = n\pi \Rightarrow x = \frac{n\pi}{3}$

When  $2\cos 2x + 1 = 0$

Then,  $\cos 2x = -\frac{1}{2}$

$$\begin{aligned} \Rightarrow \cos 2x &= -\cos \frac{\pi}{3} \\ \Rightarrow \cos 2x &= \cos \left( \pi - \frac{\pi}{3} \right) && \left[ \because \cos(\pi - \theta) = -\cos \theta \right] \\ \Rightarrow \cos 2x &= \cos \frac{2\pi}{3} \\ \Rightarrow 2x &= 2n\pi \pm \frac{2\pi}{3} \\ \Rightarrow x &= n\pi \pm \frac{\pi}{3} \end{aligned}$$

68. If  $\frac{3\pi}{4} < \alpha < \pi$ , then  $\sqrt{\operatorname{cosec}^2 \alpha + 2\cot \alpha}$  is equal to  
 (1)  $1 + \cot \alpha$       (2)  $1 - \cot \alpha$       (3)  $-1 - \cot \alpha$       (4)  $-1 + \cot \alpha$

68. (3)

We have,  $\sqrt{\operatorname{cosec}^2 \alpha + 2\cot \alpha}$   
 $= \sqrt{1 + \cot^2 \alpha + 2\cot \alpha} = |1 + \cot \alpha|$

But  $\frac{3\pi}{4} < \alpha < \pi$

$$\Rightarrow \cot \alpha < -1 \quad \Rightarrow \quad 1 + \cot \alpha < 0$$

Hence,  $|1 + \cot \alpha| = -(1 + \cot \alpha)$

69. The value of  $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$  is

- (1) 1                      (2)  $\sqrt{3}$                       (3)  $\frac{\sqrt{3}}{2}$                       (4) 2

69. (3)

$$\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \frac{\cos^2 15^\circ - \sin^2 15^\circ}{\cos^2 15^\circ + \sin^2 15^\circ}$$

$$= \frac{\cos 2 \times 15^\circ}{1} = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

70. The value of  $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$  is equal to

- (1)  $-\frac{3}{16}$                       (2)  $\frac{5}{16}$                       (3)  $\frac{3}{16}$                       (4)  $-\frac{5}{16}$

70. (3)

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$$

$$= \frac{1}{2} \sin 20^\circ \sin 60^\circ (2 \sin 40^\circ \sin 80^\circ)$$

$$= \frac{1}{2} \sin 20^\circ \sin 60^\circ (\cos 40^\circ - \cos 120^\circ)$$

$$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \sin 20^\circ \left(1 - 2 \sin^2 20^\circ + \frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{4} \sin 20^\circ \left(\frac{3}{2} - 2 \sin^2 20^\circ\right)$$

$$= \frac{\sqrt{3}}{8} (3 \sin 20^\circ - 4 \sin^3 20^\circ)$$

$$= \frac{\sqrt{3}}{8} \sin 60^\circ = \frac{\sqrt{3}}{8} \cdot \frac{\sqrt{3}}{2} = \frac{3}{16}$$

71. The minimum value of  $3 \cos x + 4 \sin x + 8$  is

- (1) 5                      (2) 9                      (3) 7                      (4) 3

71. (4)

We know that,  $-\sqrt{3^2 + 4^2} \leq 3 \cos x + 4 \sin x \leq \sqrt{3^2 + 4^2}$

$$\Rightarrow -\sqrt{25} \leq 3 \cos x + 4 \sin x \leq \sqrt{25}$$

$$\Rightarrow -5 \leq 3 \cos x + 4 \sin x \leq 5$$

$$\Rightarrow -5 + 8 \leq 3 \cos x + 4 \sin x + 8 \leq 5 + 8$$

$$\Rightarrow 3 \leq 3 \cos x + 4 \sin x + 8 \leq 13$$

72. The number of values of  $x$  in the interval  $[0, 5\pi]$  satisfying the equation  $3 \sin^2 x - 7 \sin x + 2 = 0$  is

- (1) 0                      (2) 5                      (3) 6                      (4) 10

72. (3)

$$3 \sin^2 x - 7 \sin x + 2 = 0$$

$$\Rightarrow 3 \sin^2 x - 6 \sin x - \sin x + 2 = 0$$

$$\Rightarrow 3 \sin x (\sin x - 2) - 1 (\sin x - 2) = 0$$

$$\Rightarrow (3 \sin x - 1)(\sin x - 2) = 0$$

$$\Rightarrow \sin x = \frac{1}{3} \text{ or } 2$$

$$\Rightarrow \sin x = \frac{1}{3} \quad (\because \sin x \neq 2)$$

$$\text{Let } \sin^{-1} \frac{1}{3} = \alpha, 0 < \alpha < \frac{\pi}{2}$$

Then,  $\alpha, \pi - \alpha, 2\pi + \alpha, 3\pi - \alpha, 4\pi + \alpha, 5\pi - \alpha$  are the solutions in  $[0, 5\pi]$ .

$\therefore$  Required number of solutions = 6

73. If  $2\cos^2 x + 3\sin x - 3 = 0, 0 \leq x \leq 180^\circ$ , then the value of  $x$  is
- (1)  $30^\circ, 90^\circ, 150^\circ$  (2)  $60^\circ, 120^\circ, 180^\circ$   
 (3)  $0^\circ, 30^\circ, 150^\circ$  (4)  $45^\circ, 90^\circ, 135^\circ$

73. (1)

$$\text{We have, } 2\cos^2 x + 3\sin x - 3 = 0$$

$$\Rightarrow 2 - 2\sin^2 x + 3\sin x - 3 = 0$$

$$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0$$

$$\Rightarrow \sin x = \frac{1}{2} \text{ or } \sin x = 1$$

$$\Rightarrow x = 30^\circ, 150^\circ, 90^\circ$$

74. The solution of the equation  $\sin^{10} 2x = 1 + \cos^{10} x$  is

- (1)  $x = (2n+1)\frac{\pi}{2}$  (2)  $x = n\pi$  (3)  $x = (2n+1)\frac{\pi}{4}$  (4) None of these

74. (4)

$$\text{We have, } \sin^{10} 2x = 1 + \cos^{10} x$$

Minimum value of RHS = 1 and maximum value of LHS = 1.

Therefore, solution is possible only when  $\sin^{10} 2x = 1$  and  $\cos^{10} x = 0$ . But this is not possible.

Therefore, it has no solution

75. If  $A = \sin^2 x + \cos^4 x$ , then for all real  $x$

- (1)  $\frac{13}{16} \leq A \leq 1$  (2)  $1 \leq A \leq 2$  (3)  $\frac{3}{4} \leq A \leq \frac{13}{16}$  (4)  $\frac{3}{4} \leq A \leq 1$

75. (4)

$$A = \sin^2 x + \cos^4 x = \cos^4 x - \cos^2 x + \frac{1}{4} + \frac{3}{4}$$

$$= \left( \cos^2 x - \frac{1}{2} \right)^2 + \frac{3}{4}$$

$$\text{Where, } 0 \leq \left( \cos^2 x - \frac{1}{2} \right) \leq \frac{1}{4}$$

$$\therefore \frac{3}{4} \leq A \leq 1$$

76. The number of real roots of  $3^{2x^2-7x+7} = 9$  is

76. (1) 0 (2) 2 (3) 1 (4) 4  
 (2)

Given that,  $3^{2x^2-7x+7} = 3^2 \Rightarrow 2x^2 - 7x + 7 = 2$

$\Rightarrow 2x^2 - 7x + 5 = 0$

Now,  $D = b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 5$

$= 49 - 40 = 9 > 0$

Hence, it has two real roots.

77. For all  $x, x^2 + 2ax + (10 - 3a) > 0$ , then the interval in which  $a$  lies, is

- (1)  $a < -5$  (2)  $-5 < a < 2$  (3)  $a > 5$  (4)  $2 < a < 5$

77. (2)

As we know,  $ax^2 + bx + c > 0$  for all  $x \in R$ , iff  $a > 0$  and  $D < 0$ .

$\therefore x^2 + 2ax + (10 - 3a) > 0, \forall x \in R$

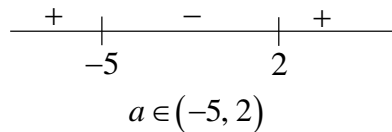
$\Rightarrow D < 0$

$\Rightarrow 4a^2 - 4(10 - 3a) < 0$

$\Rightarrow 4(a^2 + 3a - 10) < 0$

$\Rightarrow (a + 5)(a - 2) < 0$

Using number line rule



78. If the roots of the equation  $x^2 - 2ax + a^2 + a - 3 = 0$  are real and less than 3, then

- (1)  $a < 2$  (2)  $2 \leq a \leq 3$  (3)  $3 < a \leq 4$  (4)  $a > 4$

78. (1)

Given equation is  $x^2 - 2ax + a^2 + a - 3 = 0$ .

If roots are real, then  $D \geq 0$

$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$

$\Rightarrow -a + 3 \geq 0$

$\Rightarrow a - 3 \leq 0 \Rightarrow a \leq 3$

As roots are less than 3, hence  $f(3) > 0$

$9 - 6a + a^2 + a - 3 > 0$

$\Rightarrow a^2 - 5a + 6 > 0$

$\Rightarrow (a - 2)(a - 3) > 0$

$\Rightarrow$  Either  $a < 2$  or  $a > 3$ .

Hence, only  $a < 2$  satisfy.

79.  $(x - 1)(x^2 - 5x + 7) < (x - 1)$ , then  $x$  belongs to

- (1)  $(1, 2) \cup (3, \infty)$  (2)  $(2, 3)$   
 (3)  $(-\infty, 1) \cup (2, 3)$  (4) None of these

79. (3)

$$(x-1)(x^2 - 5x + 7) < (x-1)$$

$$\Rightarrow (x-1)(x^2 - 5x + 6) < 0$$

$$\Rightarrow (x-1)(x-2)(x-3) < 0$$

$$\therefore x \in (-\infty, 1) \cup (2, 3)$$

80.  $\log_2(x^2 - 3x + 18) < 4$ , then  $x$  belongs to

- (1) (1, 2)                      (2) (2, 16)                      (3) (1, 16)                      (4) None of these

80. (1)

$$\log_2(x^2 - 3x + 18) < 4$$

$$\Rightarrow x^2 - 3x + 18 < 16 \quad (\text{if } \log_a b < c \Rightarrow b < a^c)$$

$$\Rightarrow x^2 - 3x + 2 < 0$$

$$\Rightarrow (x-1)(x-2) < 0$$

$$\Rightarrow x \in (1, 2)$$

81. If  $x$  is real, then the maximum and minimum values of the expression  $\frac{x^2 - 3x + 4}{x^2 + 3x + 4}$  will be

- (1) 2, 1                      (2)  $5, \frac{1}{5}$                       (3)  $7, \frac{1}{7}$                       (4) None of these

81. (3)

$$\text{Let } y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$$

$$\Rightarrow (y-1)x^2 + 3(y+1)x + 4(y-1) = 0$$

$\therefore x$  is real

$$\therefore D \geq 0$$

$$\Rightarrow 9(y+1)^2 - 16(y-1)^2 \geq 0$$

$$\Rightarrow -7y^2 + 50y - 7 \geq 0$$

$$\Rightarrow 7y^2 - 50y + 7 \leq 0$$

$$\Rightarrow (y-7)(7y-1) \leq 0 \quad \dots(i)$$

$$\Rightarrow y \leq 7 \text{ and } y \geq \frac{1}{7}$$

$$\Rightarrow \frac{1}{7} \leq y \leq 7$$

Hence, maximum value is 7 and minimum value is  $\frac{1}{7}$ .

82. If the equations  $x^2 + 2x + 3 = 0$  and  $ax^2 + bx + c = 0$ ,  $a, b, c \in R$ , have a common root, then  $a : b : c$  is

- (1) 1 : 2 : 3                      (2) 3 : 2 : 1                      (3) 1 : 3 : 2                      (4) 3 : 1 : 2

82. (1)

Given equation are

$$x^2 + 2x + 3 = 0 \quad \dots(i)$$

and  $ax^2 + bx + c = 0 \quad \dots(ii)$

Since, Eq. (i) has imaginary roots.

So, Eq. (ii) will also have both roots same as Eq. (i).

Thus,  $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$

Hence,  $a : b : c$  is  $1 : 2 : 3$ .

83. If  $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \dots + 10(11)^9 = k(10)^9$ , then  $k$  is equal to

- (1)  $\frac{121}{10}$                       (2)  $\frac{441}{100}$                       (3) 100                      (4) 110

83. (3)

Given,  $k \cdot 10^9 = 10^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \dots + 10(11)^9$

$$\Rightarrow k = 1 + 2\left(\frac{11}{10}\right) + 3\left(\frac{11}{10}\right)^2 + \dots + 10\left(\frac{11}{10}\right)^9 \quad \dots(i)$$

$$\Rightarrow \left(\frac{11}{10}\right)k = 1\left(\frac{11}{10}\right) + 2\left(\frac{11}{10}\right)^2 + \dots + 9\left(\frac{11}{10}\right)^9 + 10\left(\frac{11}{10}\right)^{10} \quad \dots(ii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$k\left(1 - \frac{11}{10}\right) = 1 + \frac{11}{10} + \left(\frac{11}{10}\right)^2 + \dots + \left(\frac{11}{10}\right)^9 - 10\left(\frac{11}{10}\right)^{10}$$

$$\Rightarrow k\left(\frac{10-11}{10}\right) = \frac{1\left[\left(\frac{11}{10}\right)^{10} - 1\right]}{\left(\frac{11}{10} - 1\right)} - 10\left(\frac{11}{10}\right)^{10} \quad [\because \text{In GP, sum of } n \text{ terms} = \frac{a(r^n - 1)}{r - 1}, \text{ where } r > 1]$$

$$\Rightarrow -k = 10\left[10\left(\frac{11}{10}\right)^{10} - 10 - 10\left(\frac{11}{10}\right)^{10}\right]$$

$$\Rightarrow k = 100$$

84. Let  $\alpha$  and  $\beta$  be the roots of equation  $x^2 - 6x - 2 = 0$ . If  $a_n = \alpha^n - \beta^n$ , for  $n \geq 1$ , then the value of

$\frac{a_{10} - 2a_8}{2a_9}$  is equal to

- (1) 6                      (2) -6                      (3) 3                      (4) -3

84. (3)

Since,  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - 6x - 2 = 0$ .

Or  $x^2 = 6x + 2$

$\therefore \alpha = 6\alpha + 2$

$$\Rightarrow \alpha^{10} = 6\alpha^9 + 2\alpha^8 \quad \dots(i)$$

Similarly,  $\beta^{10} = 6\beta^9 + 2\beta^8 \quad \dots(ii)$

On subtracting Eq. (ii) from Eq. (i), we get

$$\alpha^{10} - \beta^{10} = 6(\alpha^9 - \beta^9) + 2(\alpha^8 - \beta^8)$$

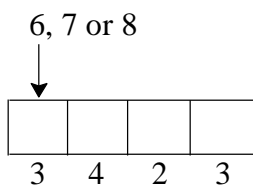


$$\begin{aligned} \Rightarrow a_{10} &= 6a_9 + 2a_8 && (\because a_n = \alpha^n - \beta^n) \\ \Rightarrow a_{10} - 2a_8 &= 6a_9 \\ \Rightarrow \frac{a_{10} - 2a_8}{2a_9} &= 3 \end{aligned}$$

85. The number of integers greater than 6000 that can be formed, using the digits 3, 5, 6, 7 and 8 without repetition, is  
 (1) 216 (2) 192 (3) 120 (4) 72

85. (2)  
 The integers greater than 6000 may be of 4 digit or 5 digit.  
 So, here two cases arise.

**Case I** When number is of 4 digit.  
 Four digit number can starts from 6, 7 or 8



Thus, total number of 4 digit number, which are greater than 6000 =  $3 \times 4 \times 3 \times 2 = 72$

**Case II** When number is of 5 digit.  
 Total number of five digit number which are greater than 6000 =  $5! = 120$

$$\therefore \text{Total number of integers} = 72 + 120 = 192$$

86. The letters of the word MODESTY are written in all possible orders and these words are written out as in dictionary, then the rank of the word MODESTY is  
 (1) 5040 (2) 720 (3) 1681 (4) 2520

86. (3)  
 Words start with D are  $6! = 720$ , start with E are 720, start with MD are  $5! = 120$  and start with ME are 120. Now the first word starts with MO is nothing but MODESTY. Hence rank of MODESTY is 1681.

87. If the sum of the first ten terms of the series  $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \dots$ , is  $\frac{16}{5}m$ , then  $m$  is equal to  
 (1) 102 (2) 101 (3) 100 (4) 99

87. (2)  
 Let  $S_{10}$  be the sum of first ten terms of the series.

Then, we have

$$\begin{aligned} S_{10} &= \left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \dots \text{ to 10 terms} \\ &= \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + 4^2 + \left(\frac{24}{5}\right)^2 + \dots \text{ to 10 terms} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{5^2} (8^2 + 12^2 + 16^2 + 20^2 + 24^2 + \dots \text{ to 10 terms}) \\
 &= \frac{4^2}{5^2} (2^2 + 3^2 + 4^2 + 5^2 + \dots \text{ to 10 terms}) \\
 &= \frac{4^2}{5^2} (2^2 + 3^2 + 4^2 + 5^2 + \dots + 11^2) \\
 &= \frac{16}{25} ((1^2 + 2^2 + \dots + 11^2) - 1^2) \\
 &= \frac{16}{25} \left( \frac{11 \cdot (11+1)(2 \cdot 11+1)}{6} - 1 \right) \\
 &= \frac{16}{25} (506 - 1) = \frac{16}{25} \times 505 \\
 \Rightarrow & \frac{16}{5} m = \frac{16}{25} \times 505 \\
 \Rightarrow & m = 101
 \end{aligned}$$

88. If  $0 \leq x \leq 2\pi$ , then the number of real values of  $x$ , which satisfy the equation  $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ , is  
 (1) 3 (2) 5 (3) 7 (4) None of these

88. (3)  
 Given equation is  $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$$\begin{aligned}
 \Rightarrow & (\cos x + \cos 3x) + (\cos 2x + \cos 4x) = 0 \\
 \Rightarrow & 2 \cos 2x \cos x + 2 \cos 3x \cos x = 0 \\
 \Rightarrow & 2 \cos x (\cos 2x + \cos 3x) = 0 \\
 \Rightarrow & 2 \cos x \left( 2 \cos \frac{5x}{2} \cos \frac{x}{2} \right) = 0 \\
 \Rightarrow & \cos x \cdot \cos \frac{5x}{2} \cdot \cos \frac{x}{2} = 0 \\
 \Rightarrow & \cos x = 0 \text{ or } \cos \frac{5x}{2} = 0 \text{ or } \cos \frac{x}{2} = 0
 \end{aligned}$$

Now,  $\cos x = 0$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad [ \because 0 \leq x < 2\pi ]$$

$$\cos \frac{5x}{2} = 0$$

$$\Rightarrow \frac{5x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \dots$$

$$\Rightarrow x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5} \quad [ \because 0 \leq x < 2\pi ]$$

and  $\cos \frac{x}{2} = 0$

$$\Rightarrow \frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$\Rightarrow x = \pi \quad [ \because 0 \leq x < 2\pi ]$$

$$\text{Hence, } x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$$

**89.** The value of  ${}^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + \dots + n \cdot {}^nP_n$  is

- (1)  ${}^{n+1}P_{n+1} - 1$                       (2)  ${}^{n+1}P_{n+1}$                       (3)  ${}^{n+1}P_{n+1} + 1$                       (4) None of these

**89.** (1)

$$\begin{aligned} & {}^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + \dots + n \cdot {}^nP_n \\ &= (1)! + 2(2)! + 3(3)! + \dots + n(n)! \quad (\because x! = {}^xP_x, x \in N) \\ &= (2-1)1! + (3-1)2! + (4-1)3! + \dots + [(n+1)-1]n! \\ &= (2!-1!) + (3!-2!) + (4!-3!) + \dots + [(n+1)!-n!] \quad [\because x(x-1)! = x!] \\ &= {}^{n+1}P_{n+1} - 1 \end{aligned}$$

**90.** Given 5 flags of different colours, how many different signals can be generated, if each signal requires the use of 2 flags, one below the other.

- (1) 18                      (2) 20                      (3) 19                      (4) 23

**90.** (2)

Number of ways to choose the first flag = 5  
 Number of ways to choose the second flag from the rest of four flags = 4  
 Hence, by FPC total number of ways =  $5 \times 4 = 20$