PART (C) : MATHEMATICS

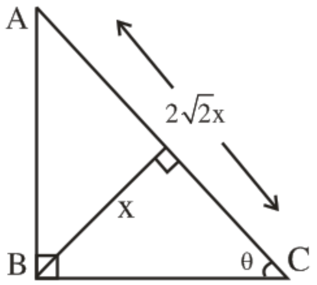
SECTION-I : (SINGLE CHOICE QUESTIONS)

This section contains **05 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

37. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are

(A) $\frac{\pi}{3} \& \frac{\pi}{6}$ (B) $\frac{\pi}{8} \& \frac{3\pi}{8}$ (C) $\frac{\pi}{4} \& \frac{\pi}{4}$ (D) $\frac{\pi}{5} \& \frac{3\pi}{10}$

37. (B)



$\sin\theta = x / BC$

$BC = x \operatorname{cosec}\theta$

$\cos\theta = \frac{BC}{2\sqrt{2}x} = \frac{\cancel{x}\operatorname{cosec}\theta}{2\sqrt{2}\cancel{x}}$

$\sin\theta\cos\theta = \frac{1}{2\sqrt{2}}$

$\sin 2\theta = \frac{1}{\sqrt{2}}$

$2\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$\theta = \frac{\pi}{8}, \frac{3\pi}{8}$

38. The number of integral solutions to the equation $|4+\log_{1/7} x| = 2 + |2+\log_{1/7} x|$ are

(A) 48 (B) 49 (C) 50 (D) 51

38. (B)

$|4+\log_{1/7} x| = |2| + |2+\log_{1/7} x|$

So, $|a+b| = |a| + |b|$

$\Rightarrow ab \ge 0$

$(2+\log_{1/7} x)(2) \ge 0$

So, $\log_{1/7} x \ge -2$

$x \le 49$

But x >0 so

$x = \{1, 2, ..., 49\}$

49 integers

39. Let p be the integral part of $\log_3 108$ and q be the integral part of $\log_5 375$ then $p+q-pq$ has the value equal to

(A) -5 (B) 5 (C) 7 (D) None

39. **(A)**

$\log_3 81 < \log_3 108 < \log_3 243$

$4 < \log_3 108 < 5$

Characteristic of $\log_3 108 = 4 = \text{p}$

$\log_5 125 < \log_5 375 < \log_5 625$

q = 3

$4+3-4(3)=7-12=-5$

40. A quadratic equation, product of whose roots $x_1 \& x_2$ is equal to 4 and satisfying the relation $\frac{x_1}{x_1-1}+\frac{x_2}{x_2-1}=2$, is

(A) $x^2-2x+4=0$ (B) $x^2-4x+4=0$

(C) $x^2+2x+4=0$ (D) $x^2+4x+4=0$

40. **(A)**

$\text{x}_1\text{x}_2-\text{x}_1+\text{x}_1\text{x}_2-\text{x}_2=2\left(\text{x}_1\text{x}_2-\left(\text{x}_1+\text{x}_2\right)+1\right)$

$\cancel{8}-\left(\text{x}_1+\text{x}_2\right)=\cancel{8}+2-2\left(\text{x}_1+\text{x}_2\right)$

$\text{x}_1+\text{x}_2=2$

Roots of $\text{x}^2-2\text{x}+4=0$

41. The sum of n terms of m A.P.s are $S_1, S_2, S_3, ...S_m$. If the first term and common difference are 1,2,3,…,m respectively, then $S_1+S_2+S_3+....+S_m=$

(A) $\frac{1}{4}mn(m+1)(n+1)$ (B) $\frac{1}{2}mn(m+1)(n+1)$

(C) $mn(m+1)(n+1)$ (D) none of these

41. **(A)**

$\text{S}_\text{r}=\frac{\text{n}}{2}\left(2(\text{r})+(\text{n}-1)\text{r}\right)$

$=\frac{\text{n}}{2}(\text{n}+1)\text{r}$

$\sum_{\text{r}=1}^{\text{m}}\text{S}_\text{r}=\frac{\text{n}(\text{n}+1)}{2}\left(\frac{\text{m}(\text{m}+1)}{2}\right)$

SECTION-II : (MULTIPLE CHOICE QUESTIONS)

This section contains **08 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE than one is/are correct.**

42. If $\sum_{r=1}^{n} r(r+1) = \frac{(n+a)(n+b)(n+c)}{3}$, where $a < b < c$, then

(A) $2b = c$

(B) $a^3 - 8b^3 + c^3 = 8abc$

(C) c is a prime number

(D) $(a+b)^2 = 0$

42. (ABC)

$$\sum r(r+1) = \frac{n(n+1)(n+2)}{3}$$

$a = 0, b = 1, c = 2$

(A) 2b = C

(B) $a^3 + (-2b)^3 + c^3 = 8abc$

(C) 'C' is a prime no.

43. In which of the following sets the inequality $\sin^6 x + \cos^6 x > \frac{5}{8}$ holds good?

(A) $\left(-\frac{\pi}{8}, \frac{\pi}{8}\right)$

(B) $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

(C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(D) $\left(\frac{7\pi}{8}, \frac{9\pi}{8}\right)$

43. (ABD)

$$\sin^6 x + \cos^6 x > \frac{5}{8}$$

$$1 - 3\sin^2 x \cos^2 x > \frac{5}{8}$$

$$\frac{3}{8} > 3\sin^2 x \cos^2 x$$

$$\frac{1}{8} > \sin^2 x \cos^2 x$$

$$\frac{1}{2} > 4\sin^2 x \cos^2 x$$

$$\frac{1}{2} > \sin^2 2x$$

$$\frac{1}{2} > \frac{1 - \cos 4x}{2}$$

$$\cos 4x > 0$$

$$\frac{\pi}{2} < 4x < \frac{\pi}{2}, \frac{3\pi}{2} < 4x < 2\pi$$

$$2\pi < 4x < \frac{5\pi}{2}, \frac{7\pi}{2} < 4x < \frac{9\pi}{2}$$

$$x \in \left(-\frac{\pi}{8}, \frac{\pi}{8}\right), x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$$

$x \in \left(\frac{7\pi}{8}, \frac{9\pi}{8}\right)$

44. $\log_2(\log_{25} 5) =$

(A) 1 (B) –1 (C) $\frac{1}{2}$ (D) 2

44. (B)

$\log_2(\log_{25} 5)$

$= \log_2\left(\frac{1}{2}\right) = -1$

45. If $\log(x^3 + 3x^2 + 2x) - \log(x+1)(x+2) = 3$, then $\log x^3 + 3\log x^2 + 2\log x =$

(A) 11 (B) 33 (C) 18 (D) None

45. (B)

$\log(x(x+1)(x+2)) - \log(x+1)(x+2) = 3$

$\log x = 3$

So

$\log x^3 + 3\log x^2 + 2\log x = 11\log x = 33$

46. $x^2 - 4$ is a factor of $f(x) = (a_1x^2 + b_1x + c_1)(a_2x^2 + b_2x + c_2)$, if

(A) $b_1 = 0,, c_1 + 4a_1 = 0$ (B) $b_2 = 0, c_2 + 4a_2 = 0$

(C) $4a_1 + 2b_1 + c_1 = 0, 4a_2 + c_2 = 2b_2$ (D) $4a_1 + c_1 = 2b_1, 4a_2 + 2b_2 + c_2 = 0$

46. (ABCD)

(A) If $b_1 = 0 \;\&\; C_1 = -b_1a_1$

Then $(a_1x^2 + b_1x + c_1)$

$= a_1(x^2 - 4)$

So $x^2 - 4$ is a free for

(B) same True

(C) $4a_1 + 2b_1 + c_1 = 0$

$x - 2$ factor of $a_1x^2 + b_1x + c_1$

$4a_1 - 2b_2 + c_2 = 0$

$x + 2$ is factor of $a_2x^2 + b_2x + c$

(D) same true

47. Let S be the set of natural numbers whose digits are drawn from $\{1,3,5,7\}$ such that no digit is repeated in any number then which of the following is (are) true? ($|S|$ = number of numbers)

(A) $|S| = 16$ (B) $|S| = 64$ (C) $\sum_{n\in s} n = 117856$ (D) $\sum_{n\in s} n = 117865$

47. (BC)

One digit nos. = 4

2 digit nos. $= 4p_2 = 4\times3 = 12$

3 digits nos. $4\times3\times2 = 24$

4 digits nos. = $4\times3\times2\times1 = 24$

Total = 64

Sum of one digit nos. $1+3+5+7=16$

Sum of 2 digit nos.

$=3(1+3+5+7)(11)$

$=33\times16$

Sum of all 3 digits nos.

$=(1+3+5+7)\times(111)\times3\times2$

$=6\times(111)\times16$

Sum of all 4 digit nos.

$=(1+3+5+7)\times(1111)\times(3\times2\times1)$

$=16(1+33+666+6666)$

117856

48. Which of the following is/are true?

(A) $\log_2 3<\log_5 17$

(B) $\log_2 24(\log_{96} 2)^{-1}-\log_2 192(\log_{12} 2)^{-1}=3$

(C) $(\log_2 5)^2>\log_2 20$

(D) $\log_{10} 5.\log_{10} 20+(\log_{10} 2)^2=1$

48. (ABCD)

(A) $\log_2 3<\log_5 17$

Then $\log_2 3^4<\log_5 17^4$

$\underbrace{\log_2 64}_{6}<\log_2 81<\underbrace{\log_2 128}_{7}$

But $17^4=83,521$

$5^7=78,125$

So, $\log_5 17^4>7$

(B) $(\log_2 24)(\log_2 96)-(\log_2 192)(\log_2 12)$

Let $\log_2 3=x$

$=(x+3)(x+5)-(x+2)(x+6)$

$=(x^2+8x+15)-(x^2+8x+12)$

= 3

(C) $(\log_2 5)^2>\log_2 20$

i.e. $\log_2 5>\log_5 20$

True $\because \log_2^5>2$ & $\log_5 20<2$

(D) $\log_{10} 5\log_{10} 20+(\log_{10} 2)^2$ let $\log_{10} 2=x$

$=(1-x)(1+x)+x^2$

$=1-x^2+x^2=1$

49. If X = 144, then
(A) no. of divisors (including 1 and X) of X = 15
(B) sum of divisors (including 1 and X) of X = 403
(C) product of divisors (including 1 and X) of $X = 12^{15}$
(D) sum of reciprocals of divisors (including 1 and X) of $X = \frac{403}{144}$

49. (ABCD)

$144 = 16 \times 9$

$= 4^2 3^2$

$= 3^2 2^4$

No. of divisors = $(3)(5) = 15$

Sum of divisors

$= (1+3+3^2)(1+2+2^2+2^3+2^4)$

$= 13(31)$

$= 403$

Product of divisors

$= (144)^{15/2}$

$= 12^{15}$

Sum of reciprocal

$$= \left(\frac{1}{1}+\frac{1}{3}+\frac{1}{3^2}\right)\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}\right)$$

$$= \left(1\left(\frac{1-\frac{1}{3^3}}{1-\frac{1}{3}}\right)\right)\left(\frac{1-\frac{1}{2^5}}{1-\frac{1}{2}}\right)$$

$$\left(\frac{31}{32}\right)\left(\frac{3}{2}\right)\left(\frac{26}{27}\right)$$

$$= \frac{403}{144}$$

SECTION-III : (INTEGER CORRECT TYPE)

This section contains **05 Questions.** The answer to each question is a **Single digit integer**, ranging from 0 to 9 (both inclusive)

50. In the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4 ……, whose n consecutive terms have the value n, then the value of $T_{150} - 10 =$ ________. (Where T_{150} is the 150th term)

50. (7)

$T_{\frac{n(n+1)}{2}} = n$

$T_{\left(\frac{n(n+1)}{2}+1\right)} = n+1$

$T_{\frac{17(18)}{2}} = 17$

$T_{153} = 17$

$T_{154} = 18$

So $T_{150} = 17$

$T_{150} - 10 = 7$

Ans. =7

51. If the set of values of 'c' for which $x^3 - 6x^2 + 9x - c$ is of the form $(x-\alpha)^2 (x-\beta), [\alpha, \beta \in R]$ are $\{a, b\}$ then $a+b$ is _________.

51. **(4)**

Roots α, α, β

$2\alpha + \beta = 6 \Rightarrow \beta = 6 - 2\alpha$

$\alpha\alpha + \alpha\beta + \alpha\beta = 9$

$\alpha^2 + 2\alpha(6 - 2\alpha) = 9$

$\alpha^2 + 12\alpha - 4\alpha^2 = 9$

$12\alpha - 3\alpha^2 = 9$

$0 = 3\alpha^2 - 12\alpha + 9$

$0 = \alpha^2 - 4\alpha + 3$

$\alpha = 1$	3
$\beta = 4$	0
$C = \alpha^2\beta$	$C = \alpha^2\beta$
$= 4$	$= 0$

$c = \{0, 4\}$

a + b = 4

52. If $\cot(\theta - \alpha), 3\cot\theta, \cot(\theta + \alpha)$ are in A.P and θ is not an integral multiple of $\frac{\pi}{2}$, then $\frac{2\sin^2\theta}{\sin^2\alpha}$ is equal to

52. **(3)**

$\cot(\theta - \alpha) + \cot(\theta + \alpha) = 6\cot\theta$

$\frac{\sin 2\theta}{\sin(\theta - \alpha)\sin(\theta + \alpha)} = \frac{6\cos\theta}{\sin\theta}$

$\theta \neq n\frac{\pi}{2}$

So we can cancel $\sin\theta$ & $\cos\theta$

$\frac{\sin\theta}{\sin^2\theta - \sin^2\alpha} = \frac{3}{\sin\theta}$

$\sin^2\theta = 3\sin^2\alpha$

$\frac{2\sin^2\theta}{\sin^2\alpha} = 3$

53. The least positive solution of $\cos 15x = \sin 5x$ is $\frac{\pi}{\lambda}$ then $\frac{\lambda}{10}$ must be.........

53. **(4)**

$\cos 15x = \sin 5x$

$= \cos\left(\frac{\pi}{2} - 5x\right)$

$15x = \frac{\pi}{2} - 5x$

$20x = \frac{\pi}{2}$

$x = \frac{\pi}{40}$ $\quad\quad\quad \lambda = 40$

54. Number of solutions of equation $\frac{2+\cos^4 x}{1+\sin^6 x} = \sin^{10} x + \cos^{10} x$ where $x \in \left[\frac{-5\pi}{2}, \frac{5\pi}{2}\right]$ is

54. **(6)**

$\frac{2+\cos^4 x}{1+\sin^6 x} = \sin^{10} x + \cos^{10} x$

$\sin^{10} x + \cos^{10} x \le \cos^2 x + \sin^2 x$

≤ 1

&

LHS is min when $\cos^4 x = 0, \sin^6 x = 1$

LHS is max. when $\cos^4 x = 1, \sin^6 x = 0$

$1 \le \text{LHS} \le 3$

So, $\therefore$ LHS = RHS = 1

i.e. $\sin^6 x = 1$ & $\cos^4 x = 0$

$\Rightarrow x = (2n+1)\frac{\pi}{2}$

$x = -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$